The effect of surface stress rotation on the Ekman pumping

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1. Introduction

Objective verification of predicted mean sea level pressure (mslp) in HIRLAM has shown a tendency for a negative mslp bias over Scandinavia in the winter season. A tendency for predicting smaller than observed cross isobaric mass flow near the surface in regions with prevailing stable stratification in the boundary layer (PBL) has also been noted. Both observations point to problems related to the parameterization of turbulence.

The direct effect of turbulent friction is to spin-down circulations in the PBL, but as a response to friction a secondary circulation is generated which spins-up the circulations in the PBL. The divergent branch of the secondary circulation, occurring immediately above the PBL, communicates the effect of PBL friction to the free atmosphere by spinning-down the part of the circulations above the PBL.

The secondary circulation associated with PBL friction has been named the Ekman pumping. In a steady state PBL the spin-down by friction is eliminated by spin-up in the Ekman pumping. Above the PBL the spin-down by the Ekman pumping must be compensated by spin-up due to other processes than friction, such as warm advection, diabatic heating or absolute vorticity advection becoming more cyclonic with height. Otherwise, a steady-state in the PBL cannot be retained. In reality, a non-steady PBL is the rule with an intensity of the Ekman pumping varying with space and time in response to spin-up or spin-down associated with various processes inside and/or outside the PBL.

In HIRLAM the calculation of turbulent surface fluxes of momentum, heat and moisture, providing lower boundary conditions for the turbulence scheme, is based on surface Rossby-number and Monin-Obukhov similarity for the steady-state, horizontally homogeneous PBL above a rigid surface. Due to an additional wave stress it has been argued (Grachev et al., 2004) that surface Rossby number similarity, implying that the surface stress is in the direction of the near-surface wind, cannot be generally valid over the ocean. Recently, this has been confirmed by observations in frontal regions over the ocean (Persson et al., 2004).

The turbulence scheme (named CBR) applied in DMI-HIRLAM is version 6.2.3 (Cuxart et al., 2000; Lenderink and Holtslag, 2004). The CBR scheme has a prognostic equation for turbulent kinetic energy (TKE) and a diagnostic, rather complex, calculation of the length scales utilized in the calculation of the shear, buoyancy, transport and dissipation terms in the TKE equation (Undén et al., Ch.3.5, 2002).

In view of the number of simplifying assumptions applied in the turbulence parameterization both related to the lower boundary conditions, including specification of the surface roughness, and to the CBR turbulence scheme, it is perhaps not surprising that the model, as noted above, has systematic errors in its simulation of the Ekman pumping.

Work has been done with the aim of reducing systematic errors related to the Ekman pumping, in particular with the emphasis on increasing the rate of filling of decaying cyclones in the model. In this context rotation of the surface stress away from the surface layer wind direction has been suggested by Tijm (2003). He investigated the effect of a clockwise turning of the surface stress by a fixed amount and found promising results for the period he studied. A tentative parameterization of the stress rotation as function of the surface layer Richardson number was suggested by Nielsen, 2004 and test results with this parameterization in 1-dimensional (1D) DMI-HIRLAM experiments against runs without this change has been reported in Sass and Nielsen (2004). In the same report, the modified formulation was shown to have the desired effect of more rapid filling in a case study of a decaying, rather small-scale, cyclone over Denmark.

In the present article we show in section 2 for an idealized, barotropic PBL that the effect of a clockwise rotation (Northern Hemisphere) of the surface stress by a constant amount relative to the near-surface wind is to increase both the intensity of the Ekman pumping and the magnitudes of the surface stress and the surface cross isobar angle. In section 3 we present results of 1D DMI-HIRLAM experiments supporting the theoretical results obtained in section 2. Finally, section 4 contains discussion and conclusions.

2. Theory

The horizontal mean momentum equations for the steady state, horizontally homogeneous barotropic PBL are

$$\begin{aligned} 0 &= f(v - v_{g0}) - \frac{\partial \overline{u'w'}}{\partial z} = f v_a + F_x && (1) \\ 0 &= -f(u - u_{g0}) - \frac{\partial \overline{v'w'}}{\partial z} = -f u_a + F_y, && (2) \end{aligned}$$

where $\vec{F} = (-\partial/\partial z(\overline{u'w'})\vec{i} - \partial/\partial z(\overline{v'w'})\vec{j})$ is the turbulent frictional force, $\vec{\tau} = (-\overline{u'w'}\vec{i} - \overline{v'w'}\vec{j})$ is the kinematic Reynolds stress, $\vec{V_{g0}} = u_{g0}\vec{i} + v_{g0}\vec{j}$ is the geostrophic wind and $\vec{V} = u\vec{i} + v\vec{j}$ is the mean wind velocity. A primed variable denotes deviation from the mean.

Vertical integration over the depth, h, of the PBL, assuming a level bottom surface and no friction at the top of the PBL yields after some manipulation (Nielsen and Sass, 2004).

$$\langle \vec{F} \rangle = f\vec{k} \times \langle \vec{V_a} \rangle = -h^{-1}\vec{\tau_s}. \tag{3}$$

$$w(h) = f^{-1}\vec{k} \cdot \nabla \times \vec{\tau_s} \tag{4}$$

$$|\langle \vec{V_a} \rangle| \cos\alpha_F = \frac{1}{fh}|\vec{\tau_s}| \cos\alpha_F. \tag{5}$$

$$F_A = f V_{g0} |\langle \vec{V_a} \rangle| \cos\alpha_F = V_{g0} |\langle \vec{F} \rangle| \cos\alpha_F, \tag{6}$$

Equation (3) shows that $\langle\vec{F}\rangle$, the mean value of the turbulent frictional force in the PBL, is in the opposite direction of the kinematic surface stress $\vec{\tau_s} = (-\overline{u'w'_s}\vec{i} - \overline{v'w'_s}\vec{j})$ and perpendicular to $\langle\vec{V_a}\rangle$, the mean value of the ageostrophic wind in the PBL.

Equation (4) shows that $w(h)$, the vertical velocity at the top of the PBL, which is a measure of the intensity of the Ekman pumping, is proportional to the vertical component of the curl of the surface stress.

In (5) $\cos\alpha_F$ is the angle between the surface stress (or -$\langle\vec{F}\rangle$) and the geostrophic wind. Since $|\langle\vec{V_a}\rangle|\cos\alpha_F$ is the mean ageostrophic wind in the direction perpendicular to $\vec{V}_{g0}$ the lhs of (5) is proportional to the net cross isobaric mass flow in the PBL. The equation shows that the net cross isobaric mass flow is proportional to the component of the surface stress along $\vec{V}_{g0}$.

Equation (6) states that the rate of work done by the mean horizontal pressure gradient force is equal in magnitude to the rate of work done by the mean frictional force.

Suppose that $\vec{\tau_s}$ is not in the direction of the near-surface wind, $\vec{V_s}$, but rotated by a constant angle $0 < |\Delta\alpha| < \alpha_0$ with respect to $\vec{V_s}$. In the Northern Hemisphere (NH) a clockwise and counter clockwise rotation of $\vec{\tau_s}$ relative to $\vec{V_s}$ increases and decreases the component of $\vec{\tau_s}$ along $\vec{V}_{g0}$, respectively. It follows from (5) that the net cross isobaric mass flow in the PBL increases and decreases correspondingly. According to (4) the increase and decrease in the net cross isobaric mass flow must be connected with an increase and decrease in $|\vec{\tau_s}|$, respectively, since $|\nabla \times \vec{\tau_s}|$ is invariant to a rotation of the surface stress. According to (3) the mean frictional force increases and decreases correspondingly.

The tendency for having a too weak 'Ekman pumping' in HIRLAM therefore might suggest a clockwise rotation in the model of $\vec{\tau_s}$ relative to $\vec{V_s}$. Such a rotation by an amount $0 < \Delta\alpha \le \alpha_0$ leads to a new equilibrium with PBL height h_*, a surface stress $\vec{\tau}_{s_*}$ having an angle α_{F_*} relative to $\vec{V}_{g0}$ and a surface layer wind with a cross isobar angle $\alpha_{0_*} = \alpha_{F_*} + \Delta\alpha$. If F_A and F_{A_*} is the rate of work done by the mean frictional force before and after the surface stress rotation, respectively, we have $F_A = V_{g0}\tau_s h^{-1}\cos\alpha_0$ and $F_{A_*} = V_{g0}\tau_s h^{-1}\cos\alpha_F = V_{g0}\tau_{s_*}{h_*}^{-1}\cos\alpha_{F_*}$. From the latter two relations and the increase in $|\langle\vec{F}\rangle|$ in response to the clockwise rotation we get

$$\frac{h_*\tau_s}{h\tau_{s_*}} = \frac{\cos\alpha_{F_*}}{\cos\alpha_F} < 1 \tag{7}$$

or, since $\alpha_{F_*} = \alpha_{0_*} - \Delta\alpha$ and $\alpha_F = \alpha_0 - \Delta\alpha$,

$$\alpha_{0_*} > \alpha_0, \tag{8}$$

showing that the near-surface wind has a larger cross isobar angle α_{0*} in the equilibrium obtained in response to the surface stress rotation. Generally, it must be expected that the response to a constant surface stress rotation angle, i.e. the adjusted equilibrium values τ_{s_*}, h_* and α_{0_*}, depends on the turbulence parameterization scheme in use. One-dimensional DMI-HIRLAM experiments with different turbulence parameterization schemes, presented in section 3, show such a dependence. The effect of a clockwise rotation of the surface stress (NH) is shown schematically in Figure 1.

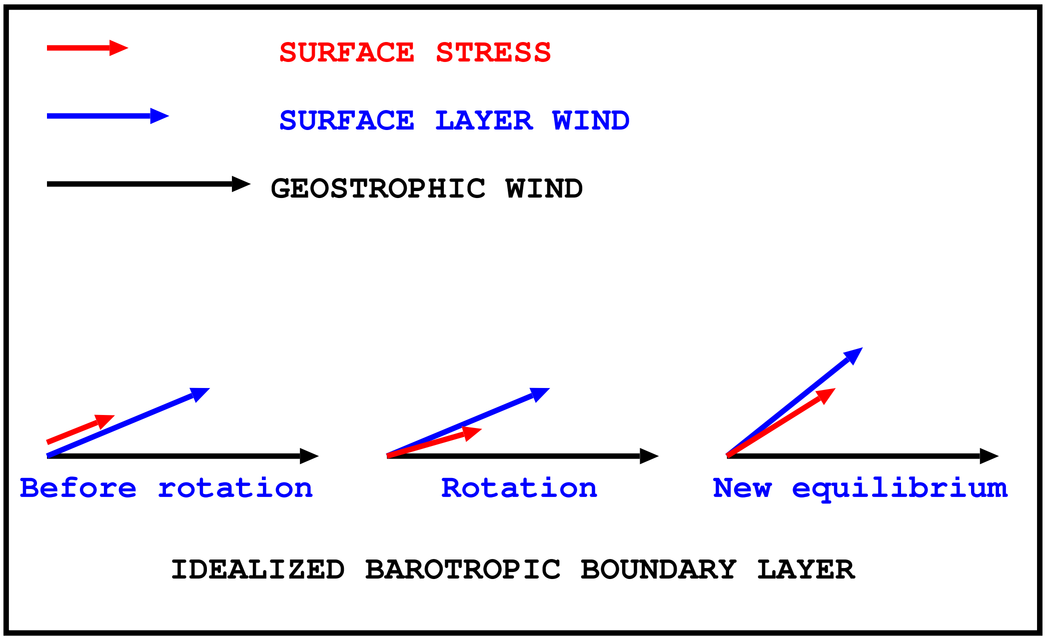

Figure 1: Schematic picture of the effect of a clockwise rotation of the surface stress in the Northern Hemisphere.

3. Experiments

A number of 1D experiments has been performed with two different turbulence parameterization schemes, named HOL and CBR. The former is a K-closure scheme (Nielsen, 1998) and the latter has turbulent kinetic energy (TKE) as a prognostic variable (Cuxart et al., 2000; Lenderink and Holtslag, 2004). To show the effect of a clockwise rotation of the surface stress relative to the surface layer wind experiments have been performed with and without surface stress rotation.

The parameterization of the surface stress rotation angle, α, was

$$\cos\alpha = 1 - \left(\frac{Ri_*}{1 + aRi_*}\right)^{\gamma}, \tag{9}$$

with $Ri_* = Ri + Ri_0$, $a = (1 - (0.5)^{1/2})^{-1/\gamma}$, $Ri_0 = 0.02$ and $\gamma = 0.5$. In the current operational DMI-HIRLAM the values of the constants have been changed to $Ri_0 = 0$, $\gamma = 1$ and $a = (1 - 0.9)^{-1/\gamma}$.

The initial conditions for the 1D runs were specified as follows: A dry, barotropic atmosphere with a constant with height relative humidity of 20% and a geostrophic wind $V_g = 10\, ms^{-1}$, a surface temperature $T_s = 10°C$, a constant lapse rate of $0.009\, Km^{-1}$ up to 1500 m followed by isothermal conditions, and a bottom surface consisting of bare land with a roughness length $z_0 = 0.01\, m$. Runs were made at latitude $70°\, N$ from initial time 00 UTC on 20 December.

Figure 2 shows the effect on the surface cross isobar angle (α_0) and the cross isobaric mass flow (cmf) of (a): the applied turbulence scheme, (b): a change in the vertical model resolution and (c): a clockwise rotation of the surface stress relative to the surface layer wind. The surface cross isobar angle is calculated as the angle between $\vec{V}_N$, the

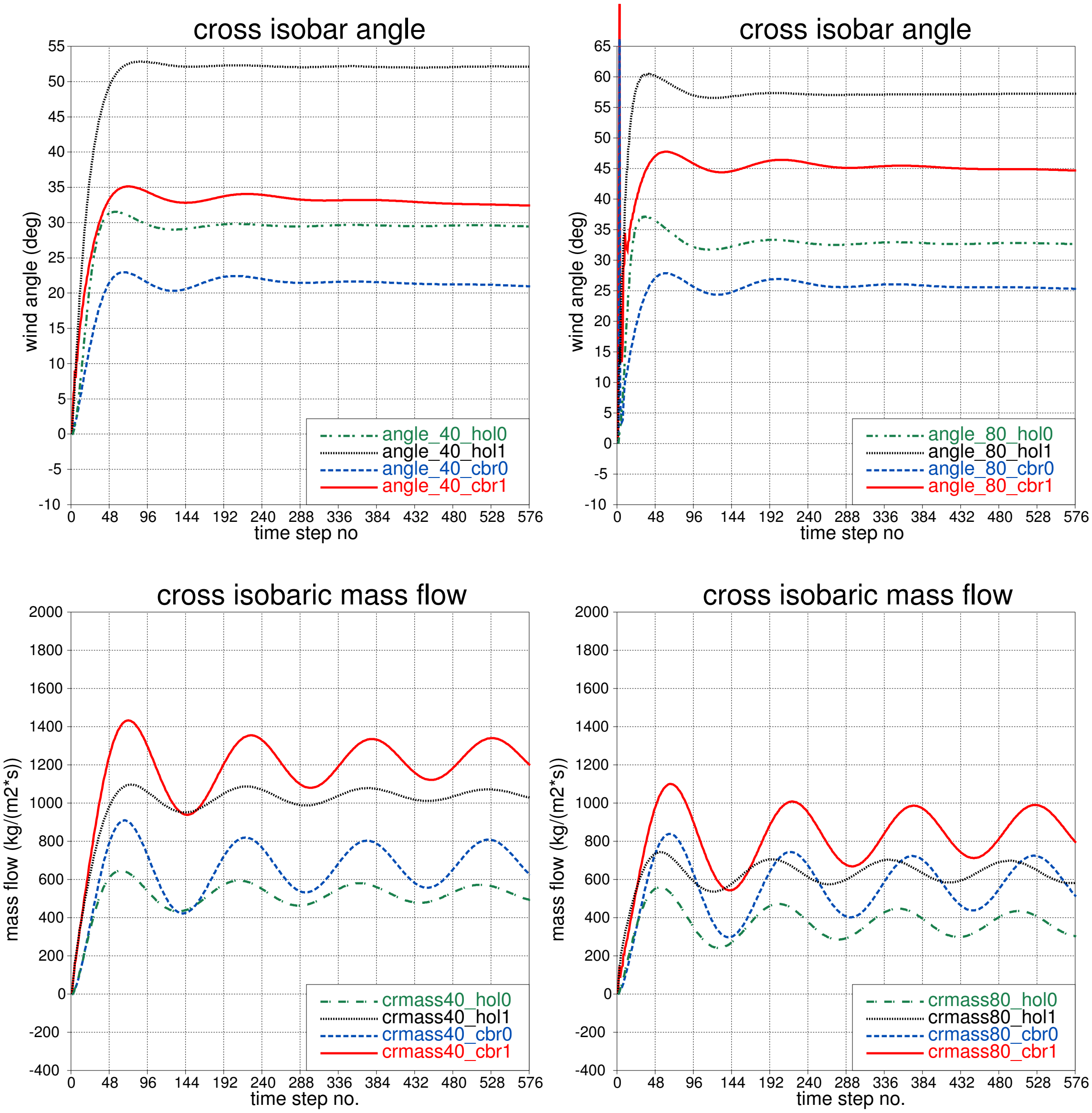

Figure 2: Variation with forecast lead time of surface cross isobar angle (top row) and cross isobaric mass flow (bottom row) in 1D-DMI-HIRLAM for barotropic conditions. Subscripts hol0 and cbr0 are for runs with the Holtslag scheme and the CBR scheme, respectively. Subscripts hol1 and cbr1 are for the same schemes with a clockwise rotation of the surface stress relative to the surface layer wind (see text). Subscripts 40 and 80 denote 40 and 80 vertical model levels, respectively. The location is at 70° N and the runs start from 00 UTC on 20 December with $z_0 = 0.01\,m$, $V_g = 10\,ms^{-1}$, $T_s = 10^\circ\,C$ (surface temperature), a lapse rate $0.009 Km^{-1}$ up to 1500 m and isothermal conditions above. The initial relative humidity is 20% and constant with height. Time step 576 corresponds to 48 hours. Note the 4 inertial cycles in the cross isobaric mass flow and the more rapid damping of the corresponding oscillation in α_0.

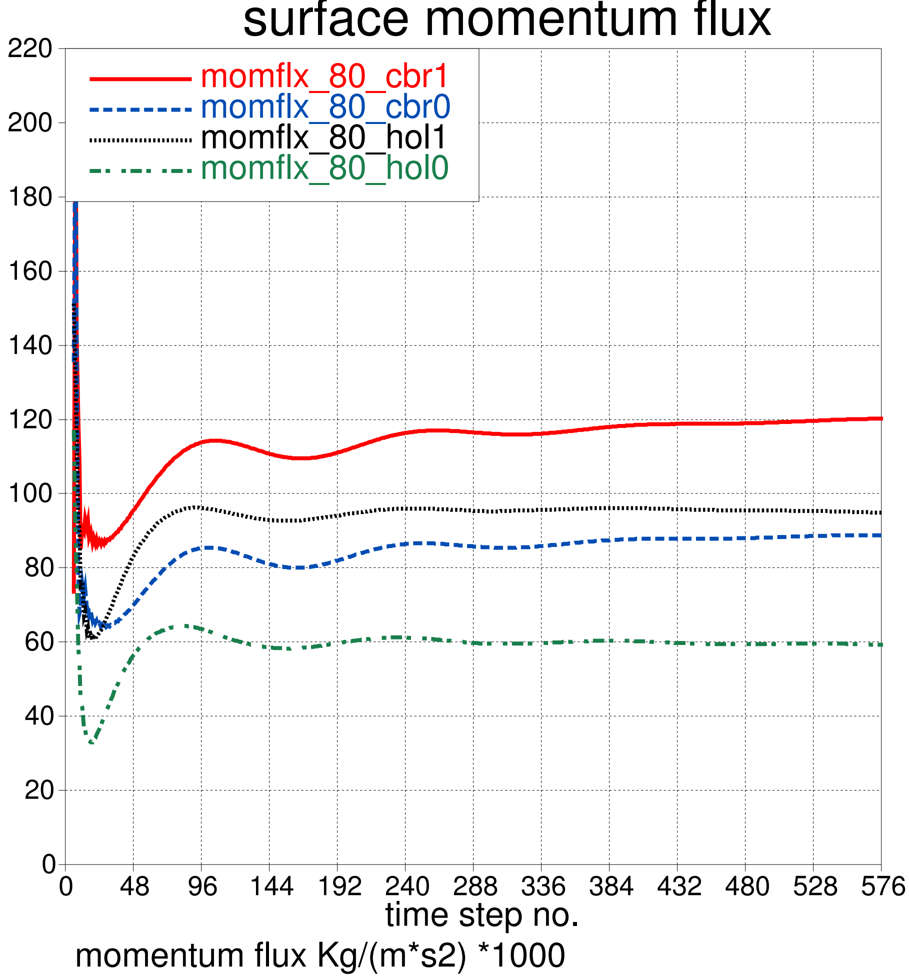

Figure 3: Evolution with time of the surface momentum flux. Initial conditions and meaning of subscripts are the same as in Figure 2.

lowest model level wind, and $\vec{V}_{g0}$. In the 40-level and 80-level versions the height of the lowest model level is approximately 35 m and 10 m, respectively.

The turbulence scheme HOL has a larger α_0 and a smaller cmf than the CBR scheme. In both schemes the response to an increase in the number of vertical levels from 40 to 80 is a small decrease in α_0 and a moderate decrease in cmf.

The response to a clockwise turning of the surface stress relative to the surface layer wind is an increase of α_0 and cmf in both schemes. In the 40 level versions the change in α_0 is largest in HOL (from about 30° to about 52°, whereas the relative increase in α_0 from 40 to 80 levels is largest in CBR. The decrease in cmf with vertical model resolution is largest in CBR.

Both with and without surface stress rotation the difference in performance prevails at very high resolution. This shows that the PBL in HOL and CBR evolves fundamentally different. Note in this context the phase lag (increasing with increasing vertical resolution) between the inertial oscillations of cmf in HOL and CBR and its (relatively weak) sensitivity to surface stress rotation.

In section 2 it was shown that a clockwise rotation (NH) of the surface stress relative to the surface layer wind should lead to a new equilibrium with an increased magnitude of the surface stress. This is confirmed by Figure 3, showing the time evolution of the magnitude of the surface momentum flux for HOL and CBR with and without rotation of the surface stress. Other figures (presented in Nielsen and Sass, 2004) show a larger increase in the PBL height in response to a clockwise rotation of the surface stress in CBR than in HOL. Without stress rotation a similar difference in response to baroclinicity was found with considerably larger variation of the PBL height with baroclinicity (e.g. from cold to warm advection) in CBR (Nielsen and Sass, 2004).

4. Discussion and conclusion

An idealized barotropic PBL over a rigid surface has been studied. It has been shown that a clockwise turning of the surface stress relative to the surface layer wind in the Northern Hemisphere intensifies the 'Ekman-pumping' by increasing the magnitude of the surface stress and the surface cross isobar angle.

The operational DMI-HIRLAM model prior to the implementation of surface stress rotation suffered from a too slow filling of surface cyclones, which was interpreted as a symptom of a weaker Ekman-pumping in the model than in the atmosphere.

In Nielsen, 2004, a parameterization of surface stress rotation was suggested with the aim of improving the performance of the Ekman-pumping in DMI-HIRLAM.

The theoretical results and 1D experiments presented in this article indicate that surface stress rotation can be used as a tool, compensating for weaknesses in the turbulence parameterization that generally results in a too weak Ekman pumping. The 1D experiments further indicate that the response to surface stress rotation in terms of magnitude of surface stress and net cross isobaric mass flow in the PBL depends on the turbulence parameterization scheme in use.

There are probably several reasons for a weaker than observed Ekman-pumping in the model. Sources of errors include: assumption of stationarity and horizontal homogeneity, parameterization of diagnostic turbulent length scales (mixing and dissipation length scales) and application of barotropic similarity relations in the surface flux calculations. It has been shown in 1D experiments (Nielsen and Sass, 2004) that the response to baroclinicity depends on the applied turbulence scheme.

The authors present view is that a good functioning turbulence parameterization scheme applied over a rigid surface has a minimum need for tuning, such as rotation of the surface stress. The presented one-dimensional experiments indicate that increased vertical resolution does not eliminate the need for optimization of the CBR scheme used in DMI-HIRLAM. Developing a more optimal turbulence parameterization might require fundamental changes of the parameterization. Until this has been achieved the suggested parameterization of surface stress rotation is considered to be an efficient (and simple) approach applicable in short range weather forecasting. The performance of stress rotation in longer range forecasting and in climate scenario runs has not been studied.

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