Recent and future developments in turbulence modeling in HIRLAM

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1 Introduction

This note describes recent and future developments in turbulence modeling in HIRLAM.It applies to versions of the TKE-l turbulence scheme (commonly loosely denoted CBR) with the new length scale formulation developed by Geert Lenderink (KNMI). This scheme will be denoted CGL. Fur-

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ed the following equation

$$\frac{}{\partial t} = \frac{\partial}{\partial z}\left(K_\phi \frac{\partial \phi}{\partial z}\right) \tag{1}$$

This is correct for a Businessq atmosphere with no density changes with height. The errors introduced by this formulation are of the order of 5 %; that is, the atmosphere looses about 5 % of its surface fluxes. In the last version of CGL we changed the formulation to

$$\frac{\partial \phi}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_\phi \frac{\partial \phi}{\partial z}\right) = \frac{\partial}{\partial p^*}\left(\rho^2 K_\phi \frac{\partial \phi}{\partial p^*}\right), \tag{2}$$

with $p^* = p/g$. Experiments showed that this formulation conserved heat, moisture and momentum

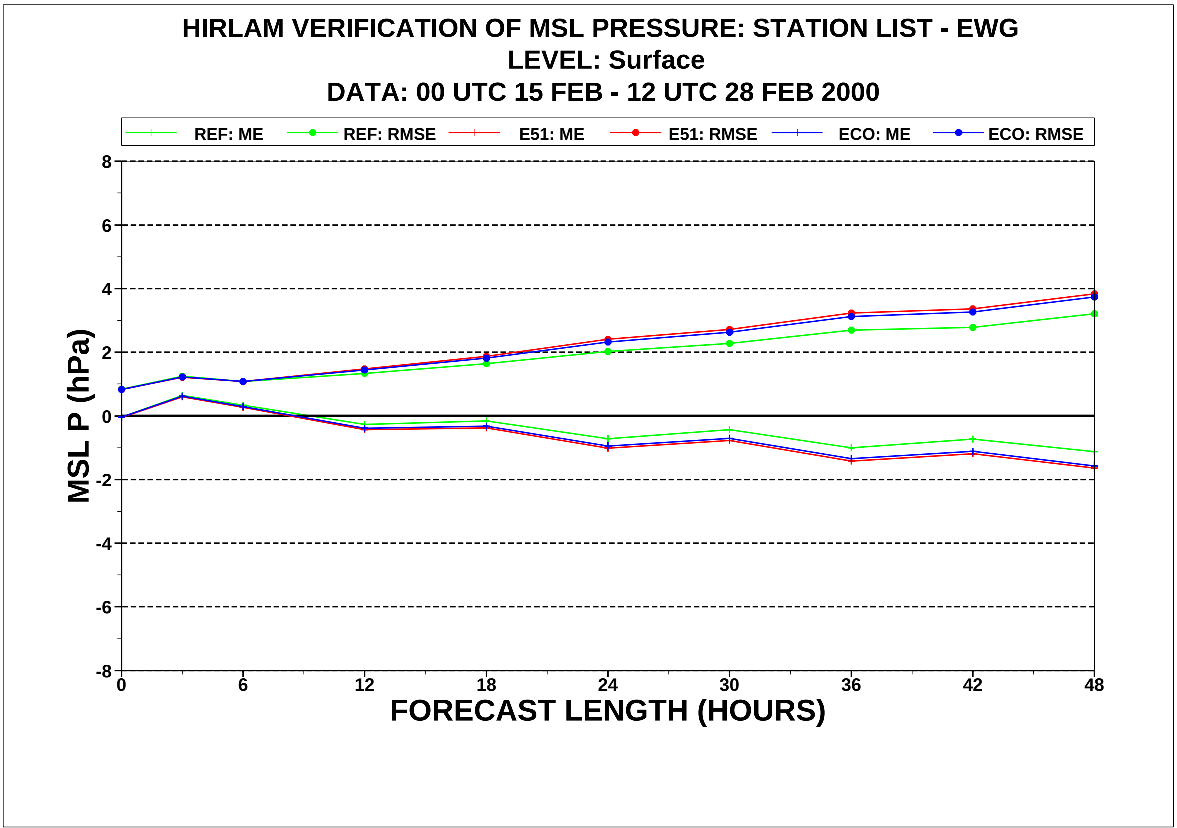

Figure 1: Pressure bias for three difference versions of the turbulence scheme: REF: CBR reference version 5.0.0; E51: CGL before the modifications described here; ECO: CGL with the modifications.

increase this surface drag, we increased the mixing length for stable conditions considerably, which from ECMWF experience (Beljaars and Viterbo, 1998) appears to be very important for the skill of operational models. We however could not increase mixing for neutral conditions, because this would lead to unrealistic wind profiles conflicting with neutral wind profile relations. These changes, together with the update in the diffusion solver as described above, led to a rather small (and a bit dissapointing) improvement in the pressure bias (run ECO in Fig. 1. It did not quite fill the gab between CBR and CGL. Artificially increasing the surface drag by a factor of two however led to a significant improvement in surface pressure bias, but (at the same time) decremented the surface wind predictions considerably. Therefore, it seems that the atmosphere needs significantly more drag from the surface to get a realistic evolution of the synoptic systems, but that - at the same time - this additional drag cannot be obtained with realistic variations of the mixing length formulation. The hypothesis is that one (or more) source(s) of drag is not represented properly at present. This problem is still unresolved, but there is hope that the inclusion of a mesoscale orographic stress scheme may lead to improvements (communication with Laura Rontu).

4 Moist turbulence

A moist turbulence scheme can represent mixing in stratiform clouds, like stratus and stratocumulus. Done properly it will give rise to realistic lapse rates (following moist adiabats) in the clouds and realistic cloud liquid water values. A moist turbulence scheme has two important differences with a dry turbulence scheme:

- mixing in moist conservative variables, e.g. total water and liquid water potential temperature.
- the effect on condensation/evaporation on the stability

In present version of CBR/CGL the first condition is fulfilled, though in a indirect sense; that is, mixing is done on potential temperature, liquid water and water vapor separately. Though this is formally not correct, and it leads to some constraints on the cloud condensation scheme, realistic values of the conserved variables can be obtained with this approach [for details see Lenderink and van Meijgaard (2001)]. This is mainly because the conserved variables are linear combinations of these separate variables, and diffusion is a linear operator.

The effect of condensation/evaporation on the stability however is not yet represented. This means that the turbulence scheme will "see" a stable cloud layer, and will suppress turbulence in the cloud. Therefore, too shallow cloudy boundary layers are predicted with too small cloud-top entrainment rates.

Experience with a moist version of CGL is obtained in RACMO (KNMI Regional Climate Model) with realistic cloud values and realistic entrainment rates (Lenderink and Holtslag, 2000). Transfer of this scheme to HIRLAM is planned.

5 Updated length scale formulation

Research to improve the length scale formulation has been done in RACMO. The main difference with the HIRLAM version is the stability parameter on which the length scale is based. In HIRLAM, in CBR and CGL, the basic length scale is based on buoyancy effects only. This leads to conflicts with the matching to flux profile relations close to the surface. The present CGL version obeys the neutral relations, but does not represent the stability dependency close to neutral optimally. [Note that the CBR reference version did not obey neutral scaling as shown in Lenderink and de Rooy (2000)]. In order to improve on the matching with flux profile relations close to the surface, in the new scheme the Richardson number is used as a main stability parameter. This enables a good matching at the surface. This scheme has been tested extensively in RACMO. Details on this scheme – description, testing in a single column model and testing in a LAM with Cabauw tower measurements – can be found in Lenderink (2002).

6 Time Integration Scheme

The time integration scheme in HIRLAM is based on a process splitting scheme. This means that all the tendencies of the separate physical tendencies (dynamics and physics) are computed independently, and added together at the end of the time step. This is not optimal for the diffusion solver as is shown below with a simple example. In HIRLAM a switch is present (ldynvd=.true.) that adds the dynamical tendencies to the fields, passes these updated values to the diffusion scheme and performs diffusion on these (unbalanced) updated fields. This partial time stepping scheme, as will be shown below, is formally correct leading to a realistic balance between dynamics and diffusion. However, it is at the cost of the stability parameters entering the length scale formulation, which are now based on unbalanced fields. The advantages of this approach are however larger than the latter disadvantage, and the use of this switch is therefore encouraged. Moreover, Sander Tijm (personal communication) showed that a instability in the near surface wind that occurred in the KNMI version of HIRLAM for storm conditions with the normal time stepping procedure, disappeared using the partial time stepping scheme.

Here we illustrate with a simple example how a process splitting scheme may lead to the wrong equilibrium balance. This occurs if the scheme contains both explicit and (over)implicit parts. In that case, the numerical solution converges to a state that is dependent on the time step. Our analysis also suggests a simple modification to the time integration scheme that removes this dependency.

For our investigations we consider the following simple equation

$$\frac{\partial y}{\partial t} = F(y) + b \tag{3}$$

where F is a linear function of y (can be a vector) and b is a constant. One may think of y being the (liquid water potential) temperature, F being the diffusive operator (turbulent mixing) and b being the temperature tendency due to radiation. This equation therefore states the time evolution of a system governed by (longwave) radiative cooling and entrainment warming caused by turbulent mixing. We are mainly interested in the (quasi) stationary state of this system, as is approached in a slowly (compared to the radiative and turbulence time scale) evolving system. The analytical and exact equilibrium stationary state of Eq. 3 is given by

$$y^{eq}_{anal} = -F^{-1}(b). \tag{4}$$

Below we investigate whether or not this stationary state is approached by time integration of the discretized system, and we assume the following format of the time integration scheme:

$$y^{n+1} = y^n + \Delta y^F + \Delta y^b, \tag{5}$$

where Δy^F is the increment due to F, and Δy^b is the increment due to the constant forcing b, and n and $n+1$ denote the previous and new full time step, respectively. This format corresponds to the format used in many numerical weather prediction models, in which the tendencies due to the different physical processes are computed in separate routines. Assuming that the integration of b can be done explicitly yields

$$\Delta y^b = b\Delta t. \tag{6}$$

However, because of numerical stability reasons (see e.g. Beljaars 1991), the increment due to F, Δy^F, has to be done (over) implicitly; that is,

$$\frac{\Delta y^F}{\Delta t} = \alpha F(y_o + \Delta y^F) + (1-\alpha)F(y_o), \tag{7}$$

where y_o is the field where F is working on, and $(y_o + \Delta y^F)$ the provisional new value of y computed by the F-scheme. The factor α is the degree to which the scheme is implicit. (If the tendency due to F is computed based on y^n only (that is, $y_o = y^n$) the procedure above is denoted by process splitting. Both the tendency due to F and b are now computed independently based on the previous (time step n) value of y. This is the scheme presently used in the SCM.)

Using that F is assumed to be a linear operator, Eq. 7 can be written as

$$\Delta y^F - \alpha \Delta t F(y_o) - \alpha \Delta t F(\Delta y^F) = (1-\alpha)\Delta t F(y_o). \tag{8}$$

This is equivalent to

$$\tilde{F}(\Delta y^F) = \Delta t F(y_o) \tag{9}$$

with

$$\tilde{F}(x) \equiv x - \alpha \Delta t F(x) \tag{10}$$

and yields the formal solution

$$\Delta y^F = \Delta t \tilde{F}^{-1}(F(y_o)) \tag{11}$$

The time integration scheme is now given by

$$y^{n+1} = y^n + \Delta t \, (b + \tilde{F}^{-1}(F(y_o)). \tag{12}$$

In equilibrium there is no change in y from one time step to the next; that is, $y^{n+1} = y^n$. Using this in Eq. 12 and using the definition of $\tilde{F}$ (in Eq. 10) yields:

$$\tilde{F}(-b) = -b - \alpha \Delta t F(-b) = F(y_o) \tag{13}$$

If we express y_o in terms of y^n plus an update $(\Delta y)_u$,

$$y_o = y^n + (\Delta y)_u, \tag{14}$$

and using that the discretized system has converged to a stationary state for which $y^n = y^{n+1} \equiv y^{eq}_{num}$, we get for this state

$$y^{eq}_{num} = y^{eq}_{anal} + \alpha b \Delta t - (\Delta y)_u. \tag{15}$$

In this derivation we used in Eq. 13 that F is a linear operator and inserted the analytical equilibrium solution y^{eq}_{anal} for $-F^{-1}(b)$.

From Eq. 15 it is clear that with a process splitting scheme, that is $(\Delta y)_u = 0$, a time step dependent equilibrium solution is obtained. A simple way to cure this time step dependency is obtained by taking $(\Delta y)_u = \alpha b \Delta t$. In that case the equilibrium solution y^{eq}_{num} obtained by time integration equals the analytical equilibrium solution. This means that, in order to get the correct equilibrium solution by time integration, the turbulent diffusion should work on the temperature field that is updated with $\alpha b \Delta t$. Note that the overimplicit factor occurs in this update.

Another way to circumvent this time-step dependency is used in the ECMWF model. For the radiative-diffusive equation, this "fractional steps" procedure first computes the radiative tendency explicitly. This radiative *tendency* is then passed to the diffusion routine, and the diffusion routine now solves the whole system *includin* the radiative tendency implicitly (Beljaars personal communication).

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