Manipulations to determine the hybrid coordinate in HIRLAM

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1 Introduction

The pressures of the model levels are defined as:

$$p_{k+1/2} = A_{k+1/2} + B_{k+1/2}p_s$$

The hybrid coordinate system is defined as:

$$\eta_{k+1/2} = \frac{A_{k+1/2}}{p_r} + B_{k+1/2}\frac{p_r}{p_r}$$

We want to define the values of A and B to have the highest resolution in pressure (or height) in the areas of interest, normally in the boundary layer. Still, the variation of layer thickness should be smooth in order not to introduce unnecessary vertical discretisation errors. This is done here through two polynomial representations; the first one for the values of η itself, then the second one for the values of B as a funtion of level number.

The η values are determined as to be as close as possible to some user defined target resolution, a specified list of level values that one would aim towards.

2 Determining the η values

The distribution of η-levels is represented as a fourth degree polynomial. This is a compromise between the flexibility of resolution and reducing the risk of non-monotonicity.

$$\eta_{k+1/2} = \gamma_1 + \gamma_2 S_k + \gamma_2 {S_k}^2 + \gamma_3 {S_k}^3 + \gamma_4 {S_k}^4$$

where $S_k = \frac{k}{NLEV}$. The condition $\eta_{1/2} = 0$ gives

$$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 = 0$$

$$\eta_{k+1/2} = \gamma_1 + \gamma_2 S_k + \gamma_2 {S_k}^2 + \gamma_3 {S_k}^3 + (1 - \gamma_1 - \gamma_2 - \gamma_3){S_k}^4$$

The tree remaining degrees of freedom are determined by a least square fit towards the desired target distribution of levels:

$$E = \sum_{k=1}^{NLEV-1} (\eta^{target}_{k+1/2} - \eta_{k+1/2})^2$$

$$\frac{\partial E}{\partial \gamma_1} = \sum_{k=1}^{NLEV-1} 2(\eta_{k+1/2}^{target} - \eta_{k+1/2})(S_k^1 - S_k^4) = 0$$

$$\frac{\partial E}{\partial \gamma_2} = \sum_{k=1}^{NLEV-1} 2(\eta_{k+1/2}^{target} - \eta_{k+1/2})(S_k^2 - S_k^4) = 0$$

$$\frac{\partial E}{\partial \gamma_3} = \sum_{k=1}^{NLEV-1} 2(\eta_{k+1/2}^{target} - \eta_{k+1/2})(S_k^3 - S_k^4) = 0$$

This system of 3 linear equations in γ_1, γ_2 and γ_3 is solved by Gaussian elimination.

3 Determining the A:s and B:s

The values of B are specified as to follow a 5th degree polynomial in level numbers:

$$B_{k+1/2} = \sum_{n=1}^{5} \beta_n (S_k)^n$$

The top two levels are usually set to be pure pressure levels :

$$B_{1+1/2} = B_{2+1/2} = 0$$

and the lowest half level should be the surface pressure

$$B_{NLEV+1/2} = 1$$

The other two conditions are obtained through specifying the ratio (C=2) between the thickness of the model layers near the surface and in the middle of the atmosphere (p_m), e.g. 500 hPa. Such a relationship is desirable to control the thickness of the model boundary layer levels over some of the highest orography in the area.

$$(A_{k+1/2} - A_{k-1/2}) + (B_{k+1/2} - B_{k-1/2})p_r = \\ C((A_{k+1/2} - A_{k-1/2}) + (B_{k+1/2} - B_{k-1/2})p_m)$$

using the relationship between A and η

$$A_{k-1/2} = \eta_{k-1/2}p_r - B_{k-1/2}p_r$$

gives a recursive relationship for the B-value of the next half level

$$B_{k-1/2} = \frac{(1-C)(A_{k+1/2} - \eta_{k-1/2}p_r) + B_{k+1/2}(p_r - Cp_m)}{C(p_r - p_m)}$$

Then the next $A_{k-1/2}$ is given by

$$\eta_{k-1/2}p_r - B_{k+1/2}p_r$$

This is applied for the two lowest levels, $k = NLEV$ and $NLEV - 1$

Now the linear system of 5 equations can be solved :

$$\begin{aligned}
\sum_{n=1}^{5} \beta_n (S_1)^n &= 0 = (B_{1+1/2}) \\
\sum_{n=1}^{5} \beta_n (S_2)^n &= 0 = (B_{2+1/2}) \\
\sum_{n=1}^{5} \beta_n (S_{NLEV-2})^n &= B_{NLEV-1-1/2} \\
\sum_{n=1}^{5} \beta_n (S_{NLEV-1})^n &= B_{NLEV-1/2} \\
\sum_{n=1}^{5} \beta_n (S_{NLEV})^n &= 1 = (B_{NLEV+1/2})
\end{aligned}$$

In practise, it is still difficult to fit both the top and bottom of the atmosphere (the stratosphere versus the boundary layer).

A further improvement can be made by using the following approach:

1. Fit the bottom of the atmosphere and determine η_{low} values (as in Section 2).
2. Fit the top of the atmosphere and determine η_{high} values (also as in Section 2).
3. Combine the two :
$\eta = \eta_{low}\eta_{high} + (1 - \eta_{low})\eta_{high}$
4. Determine A:s and B:s as above, in this Section, from the combined η values.

4 Results

The above procedure has been used to derive values of A and B for higher vertical resolutions than the Reference 31 levels. The first example shows (Fig. 1) the layer thicknesses in pressure or height (for a scale height of 7000 m approx. eq. to a constant temperature of 240 K) as function of pressure for the Reference 31 levels and a 40 level set. There is considerably better resolution in the boundary layer and also some in most of the troposphere, while the for the stratosphere it is almost the same. The top level has been kept at 10 hPa in this set and all the following ones. Going above 10 hPa requires special care with respect to data assimilation and making sure that adequate sonde and particularly satellite data are available and used at those levels.

The next figure (Fig. 2) compares the 40 level set determined here with the 40 levels used in the ECMWF EPS system (and coinciding with the ECMWF 60 levels in the lower troposhere). the main difference is that the ECMWF lowest level is at 10 m with some very thin bottom layers, whereas we have decided to stick to about 30 m for the lowest level and have a less radical variation of thicknesses. The concept of blending height is a bit doubtful, if it is satisfied at 30 m over hetereogeneous surfaces and certainly it is not at 10 m. The other thing of concern is the discretisation of the very thin layers and its numerical effect.

Finallly, we have also derived a 50-level set and a 60 level one. The Figures 3 and 4 compare these with the above 40-level set. The boundary layer and troposheric resolutions are increased in each step, from 40 -50 - 60 levels while maintaining the stratospheric resolution.

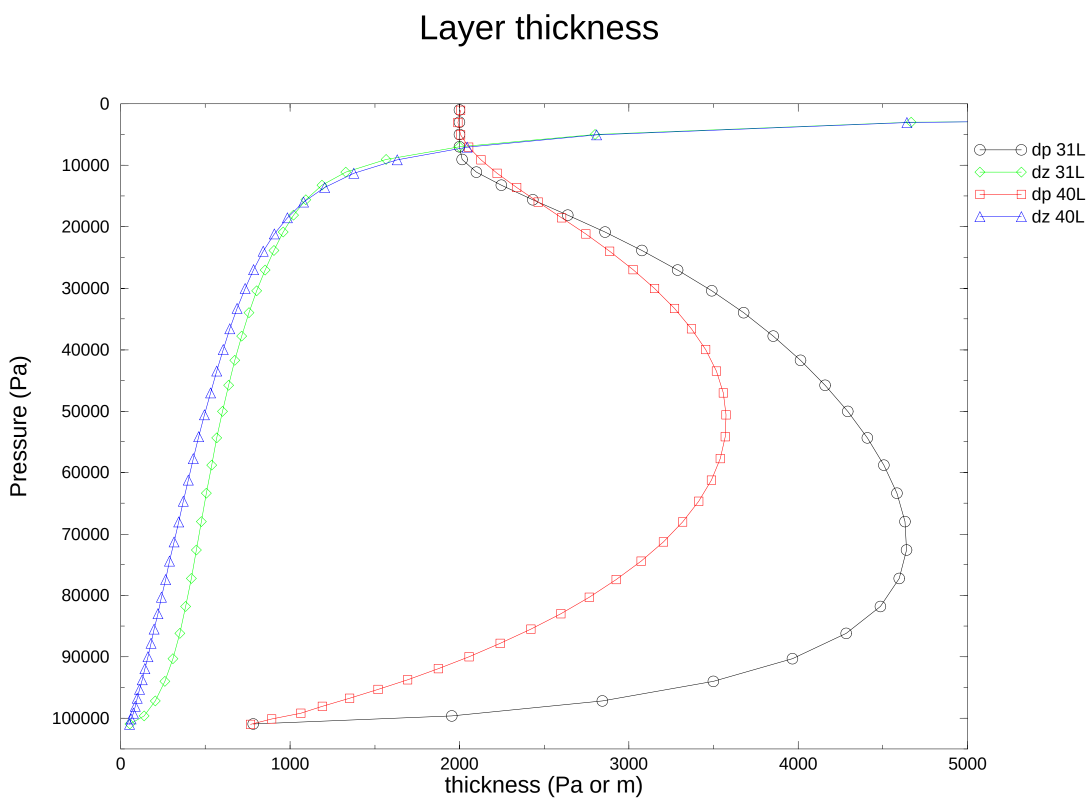

Figure 1: Layer thicknesses for 31 and 40 levels (Pa and m) as a function of pressure.

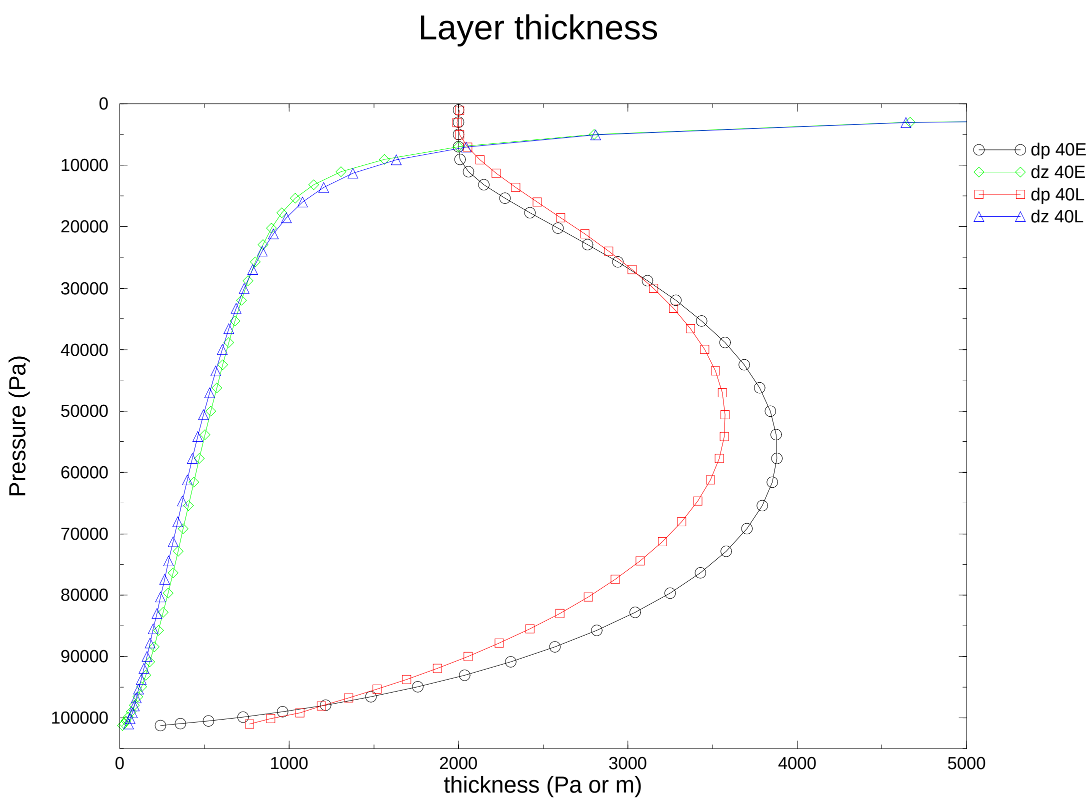

Figure 2: As Fig. 1 but comparing the 40 level set with the ECMWF EPS 40 level set.

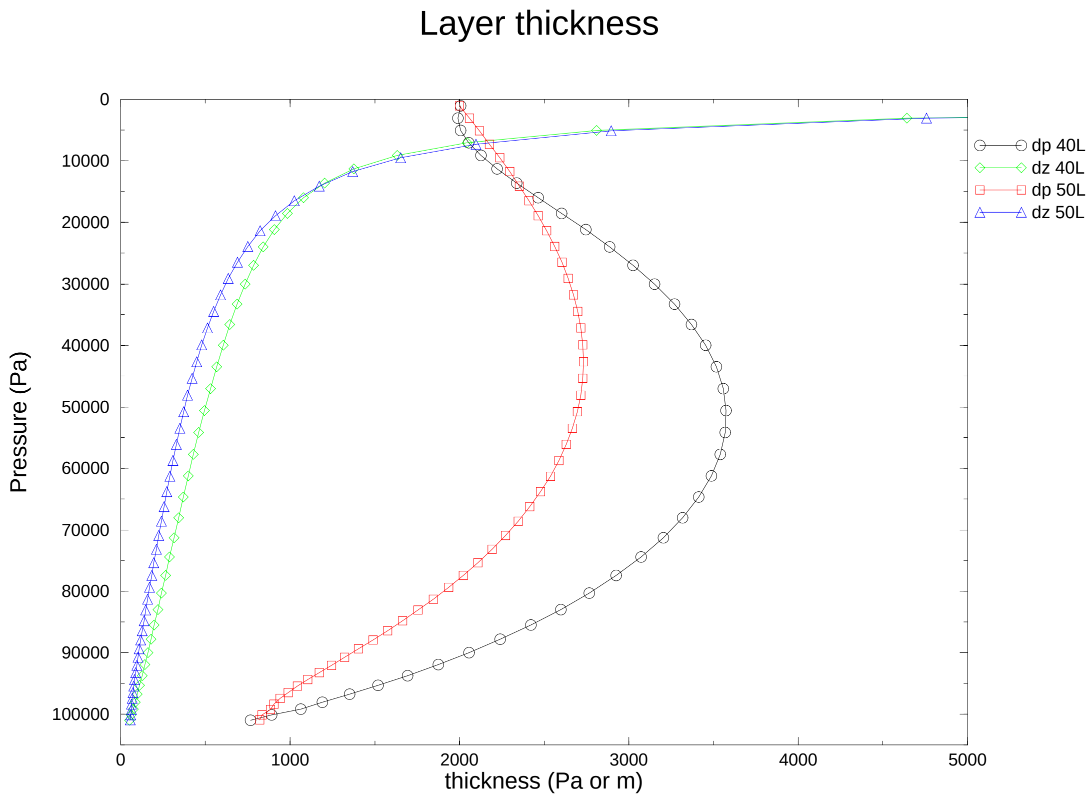

Figure 3: As Fig. 1 but comparing the 40 and 50 level sets.

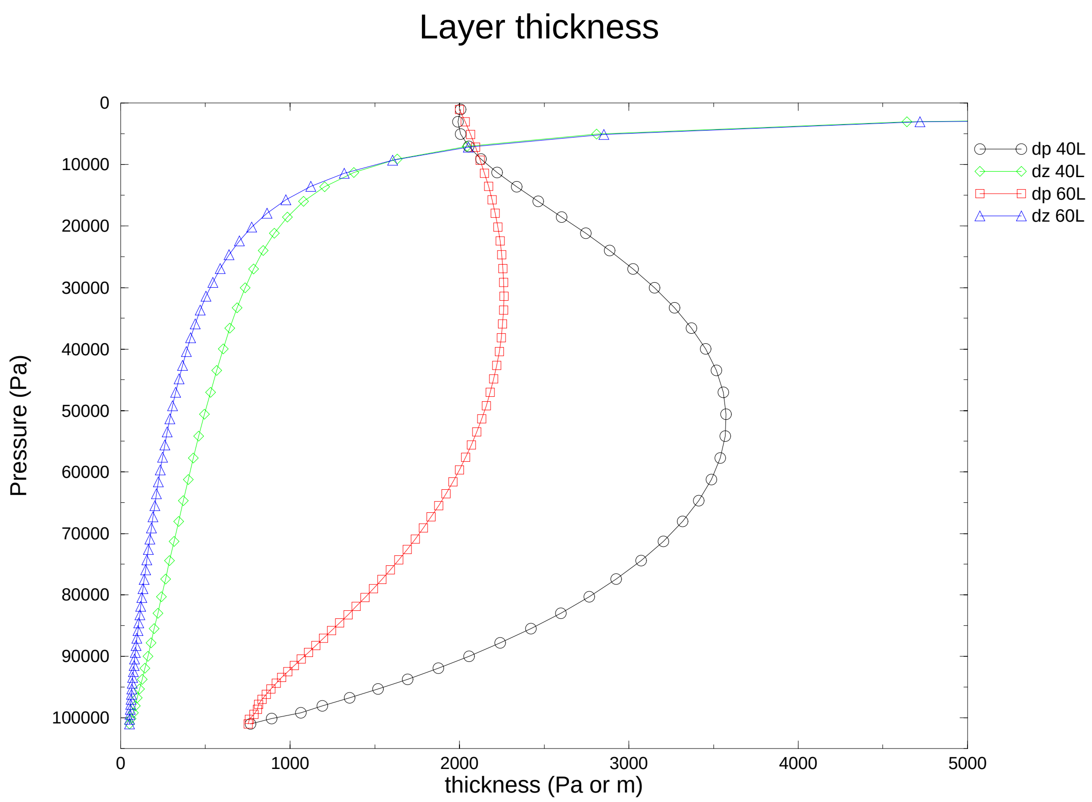

Figure 4: As Fig. 1 but comparing the 40 and 60 level sets.