TESTING ALTERNATIVE LATERAL BOUNDARY STRATEGIES: A PROGRESS REPORT.

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1. BACKGROUND.

In late 1995 the HIRLAM group decided to implement a programme re-examine the treatment of the artificial lateral boundaries of their limited area model with a view to improving it. That treatment consists of defining a 'relaxation zone' next to the boundary within which all of the fields are relaxed toward the externally supplied 'host' model coarse mesh fields; see Davies (1976). On the boundary line itself the host model fields are imposed. Within the relaxation zone these fields are combined with the fields of the fine-mesh inner area (the 'guest' area) in such a way that the noise generated by the over-specification on the boundary is damped, while simultaneously the meteorological information is allowed to enter the area with minimum distortion. This is to some extent a balancing act, which is achieved by a careful choice of (a) the relaxation parameters and (b) the width of the relaxation zone; see Kållberg (1977). The perceived strengths and weaknesses of the flow relaxation scheme can be summarised as follows.

Strengths.

(a) Stable. (b) Robust. (c) Easy to implement. (d) Computationally inexpensive.

Weaknesses.

(e) There is evidence that it can cause mass loss or gain; see McDonald (1998). (f) At points at which the characteristics are pointing out of the domain of integration we are imposing the host model solution when we do not need to. Thus we may introduce unnecessary errors when these fields are incorrect (as they will be in an operational environment). (g) The host and guest model fields will both normally be in approximate geostrophic balance. The scheme destroys this balance throughout the the boundary relaxation zone when the host and guest model velocities differ; see Cats and Åkesson (1983). (h) There are undefined parameters. The width of the boundary relaxation zone is undetermined. Let us assume its width is $(N+1)\Delta x$. Then there are N undefined parameters which must be determined either 'experimentally', as in Kållberg (1977) or by using plausibility arguments as in Davies (1983) or Lehmann (1993). It is probable that these parameters will have to be re-tuned for finer grids. (i) It has not been built from solid mathematical foundations: we are no longer solving the equations of motion, but a new set of equations with very different mathematical properties.

The first step in the program to improve the HIRLAM boundary treatment was to examine what others were doing within the Meteorological community. This review of the meteorological literature resulted in McDonald (1997) making non-radical recommendations on possible improvements to the HIRLAM lateral boundary treatment. Briefly, these were as follows.

1. Use the Baumhefner and Perkey (1982) experimental set-up to test the accuracy of the lateral boundary treatment exclusive of other complications.
2. Use host model orography in the relaxation zone.
3. Use weak forcing in the lower layers of the atmosphere in the relaxation zone.
4. Tune the relaxation coefficient for fine grids.
5. Test the NCEP boundary treatment in the HIRLAM model.
6. Test initializing the host model fields.
7. Test the stretched coordinate version of HIRLAM.

The next step in the program to improve the HIRLAM boundary treatment was to examine what scientists in other disciplines were doing. (They have invented a variety of methods for making boundaries 'as transparent as possible'. The literature is vast; see the review Tsynkov, 1998). The outcome was that in McDonald (2002a) some of these ideas were applied to the linearized shallow water equations, and a hierarchy of transparent boundary conditions was derived for which some of the above mentioned weaknesses of the relaxation scheme are eliminated.

These new boundary conditions were tested for integrations of the linearized shallow water equations in McDonald (2002a). The results were encouraging enough to continue testing in a more demanding environment, using the full shallow water equations and realistic meteorological data. In section 2 we briefly outline the results of these new tests.

2. TESTING USING THE FULL SHALLOW WATER EQUATIONS.

Promising results were obtained by McDonald (2002a) for the *linearized* shallow water equations using imposed *analytical* boundary fields. McDonald (2002b) injected more operational realism in two ways: first by restoring the non-linear terms in the shallow water equations, and secondly by imposing pseudo-operational boundary fields. This section provides a brief description of the methodology and results.

To test the boundary strategies the program of Baumhefner and Perkey (1982) was implemented: a large area host model integration generates the boundary fields for a smaller guest area which is nested within the host area. The large area fine grid forecast defines the 'correct forecast' which the the guest area fine grid forecast strives to attain. The large area coarse grid forecast furnishes the boundary fields for the guest area integration.

The experimental set-up was as follows. The host area was horizontally discretized in two different configurations on a rotated latitude-longitude grid for which $\theta_0 = 60°$. The fine mesh contained 202×178 points with $\Delta\lambda = \Delta\theta = 0.4°$. The coarse mesh contained 68×60 points with $\Delta\lambda = \Delta\theta = 1.2°$, for which every third point coincided with a fine mesh point.

The guest area consisted of 60×60 mesh points which coincided with the following host area fine grid points: $(72 \rightarrow 131)$ in the east-west direction, and $(60 \rightarrow 119)$ in the north-south direction. In round numbers the host area was $9000km \times 8000km$ and the guest area $2500km \times 2500km$

To generate a 'correct forecast' the shallow water equations were integrated for 48h on the fine grid over the host area using a time step of 10min. All fields were stored at every time step at every grid point over the guest area. This data set represents the 'correct forecast'. To generate 'coarse' boundary fields for the guest area the above procedure was repeated on the coarse grid. (This data set also represents the worst case scenario for the guest forecasts. If they do not beat the coarse mesh host forecasts they are failures.)

The forecasts were verified over a central area consisting of the following 20×20 grid points of the guest area: points $(21 \rightarrow 40)$ in the east-west direction, and points $(21 \rightarrow 40)$ in the north-south direction. All forecasts were verified against the host model fine mesh forecast.

The fine grid starting analysis for the host area consisted of a set of 500hPa height and wind fields. These were interpolated from an analysis performed using the optimal interpolation analysis of the HIRLAM, (Källén, 1996), which uses hybrid coordinates in the vertical. As a result, the wind and height fields are not in balance. To correct this a digital filter initialization (Lynch, 1996) was performed. The host area coarse grid starting analysis consisted of the fields defined at every third point of the fine grid analysis. The guest area starting fields were copied from the initialized host area fine grid fields, since the grids coincide.

In an operational context the fields from the host area coarse integration are available only intermittently to the guest model. To model this the fields from the coarse mesh integration were supplied to the guest model boundaries at intervals which are multiples of the time step: at times $m\Delta t, 2m\Delta t, 3m\Delta t$, and so on. A linear interpolation was performed to generate these fields at each time step, $n\Delta t$, between time steps $pm\Delta t$ and $(p+1)m\Delta t$. In the tests the time step was $\Delta t = 10$min, and the refreshment interval was 3h; thus $m = 18$.

In order to compare the new boundary strategies with that used operationally in the HIRLAM a 'passive buffer zone' of two lines external to the boundary line was used in the guest model integrations. Thus, in the East-West direction all the fields on lines 1, 2, 59 and 60 were refreshed using the time interpolated host model data. Lines 3 and 57 were the boundary lines. The same applied in the North-South direction.

Five boundary strategies were tested, three of which were 'well posed' in the sense that two fields were imposed at inflow and one at outflow.

Strategy (i). On the boundary line Φ was imposed at all points and v_T at the inflow points: 'Φ b.c.'.

Strategy (ii). On the boundary line $\Phi - \sqrt{\Phi_0}v_N$ was imposed at all points and v_T at the inflow points: 'characteristic b.c.'.

Strategy (iii). $\Phi - \sqrt{\Phi_0}v_N$ and v_T were imposed at the inflow points and $d(\Phi - \sqrt{\Phi_0}v_N)/dt$ was imposed at the outflow points on the boundary line: 'first order transparent b.c.'.

It must be emphasised that no field from the passive buffer zone was used in any of these well-posed strategies. Fields external to the boundary line were always estimated by extrapolation from the interior. In the final two boundary strategies the fields in the passive buffer zone were always used when needed. No extrapolations from the interior were performed.

Strategy (iv). On the boundary line Φ and v_T were imposed at all points, and the fields in the passive buffer zone were used whenever needed: ' S.L.b.c.'.

Strategy (v). This boundary treatment consisted of strategy (iv) plus relaxation of the fields toward the host model fields in the relaxation zone. Lines 3-10 and lines 50-57 in both the East-West and North-South directions define the relaxation zone : 'relaxation b.c.'.

Four aspects of the integrations were used to judge the quality of the boundary strategies. The first was the behaviour of the absolute value of the divergence, averaged over the guest integration area, $\overline{|D|}$. This is a good measure of whether the integration is well-balanced. Large values of $\overline{|D|}$ indicate a surplus gravity-inertia waves. Increasing $\overline{|D|}$ is a sign of an instability. The second aspect was the evolution of Φ averaged over the integration area. This measures mass conservation. If the mass mis-behaves then the boundary strategy is flawed. The third and fourth aspects of the integration examined were the evolution of the rms wind and height errors for the verification area, using the host model fine mesh forecast as the 'correct forecast'. The results of the nested experiments carried out in McDonald (2002b) can be summarised as follows.

1. For all five boundary strategies $\overline{|D|}$ was similar in size to that of the correct forecast and not increasing as the forecast proceeded. We can infer that the forecasts were stable and balanced.

2. The mass evolution of the 'first order transparent b.c.' forecast exhibited strange behaviour between 18h and 36h. For the four other boundary strategies the mass was well behaved, though not exactly conserved.

3. For the wind errors the boundary strategies called 'Φ b.c.' and 'first order transparent b.c.' exhibited skill out to approximately 24h. The other three showed skill out to approximately 30h. Subsequently, all except 'Φ b.c.' agreed approximately with the host model coarse mesh forecast verified over the same area. This asymptotic behaviour is reasonable since the distance from the boundary to the centre of the area is approximately 1200km. To travel this distance in 24h one must move at a speed of 14m/s, a modest advection speed. Thus it is not surprising that the forecast from approximately 24h onwards was dictated by the boundaries. Of the five boundary strategies 'characteristic b.c.' looked best overall. 'Φ b.c.' was a clear loser. Also, 'relaxation b.c.' , the strategy we use in the HIRLAM model, was worst in the 9h to 18h forecast period. Disappointingly, and in contrast to our tests using the linearized shallow water equations, 'first order transparent b.c.' showed no advantage over 'characteristic b.c.'. Lastly, 'S.L. b.c.' was quite competitive after an initial 'error shock'.

4. This small 'error shock' at the start of the forecast was more pronounced in the height errors. It developed in about 1.5h to 3h, depending on the boundary strategy. A time of 1.5 hours is consistent with a gravity wave of speed $\sqrt{\Phi_0} = 241$m/s travelling 1200km from the boundary to the centre of the area. Baumhefner and Perkey (1982) saw a similar, but much more severe effect: 'forecasts made by the unbounded coarse mesh model are more accurate than the limited area fine mesh model early in the forecast'. They explained this effect as being due to the 'excitation of rapidly moving spurious waves'. Subsequent to the shock 'first order transparent b.c.' showed essentially no skill. The strategy called 'Φ b.c.' had some skill out to about 18h, but subsequent to that showed suspect behaviour. The other 3 boundary strategies showed skill out to between 18h and 30h, with 'characteristic b.c.' again being best.

We may speculate that 'error shock' seen in the height error came from some residual imbalance in the fields. This opinion was re-enforced by repeating the experiments described above, but now using uninitialized data. The result was that all of the forecasts for all boundary strategies, although stable, had essentially no skill.

3. DISCUSSION.

In a meteorological context, non-linear effects are of primary importance. However, the analysis in McDonald (2002a) which lead to transparent boundary conditions was based on the *linearized* shallow water equations. Thus an important question was not addressed by this analysis: will the non-linear terms cause explosive growth at the boundary? Given this weakness in our theoretical guidance it is essential that we test these schemes using non-linear equations in simplified systems rather than trusting the analysis and experiments with linear systems to provide sufficient encouragement to jump straight to multi-level models. The results briefly described in section 2 reinforce this opinion.

As so often happens, expectations raised by tests performed on linear systems have to be lowered when these tests are repeated in the more challenging non-linear environment. The following interpretation of the experiments described in section 2 is reasonably fair. The boundary strategy called 'characteristic b.c.' is the best. Interestingly, the next best is 'S.L.b.c.'. Both beat the 'relaxation b.c.', which corresponds to our operational HIRLAM boundary treatment. The strategy called 'Φ b.c.' behaves unacceptably. We had expectations from McDonald, 2002a that 'first order transparent b.c.' would be superior to 'characteristic b.c.'; in fact, the former is disconcertingly badly behaved. Holstad and Lie (2001) also found the 'first order transparent b.c.' to be inferior. We can only speculate that our analysis, which included neither non-linear effects nor Rossby waves, gives poor guidance for the non-linear equations.

A number of issues need to be resolved before we consider testing 'characteristic .b.c.', the best boundary strategy, in a multi-level model. The first question is: is it robust? We can partially answer this question by including orography in the shallow water model. This I plan to do next. The second question is: what is causing the 'error shock' seen in the height errors, and how can we eliminate it?

In an operational meteorological context the imposed boundary fields will always contain errors. We would like to separate these into 'unavoidable' and 'potentially avoidable' errors. The source of the unavoidable errors is the fact that the host model supplies *forecast* fields whose accuracy, by definition, deteriorates as the forecast proceeds. The source of the potentially avoidable errors is the incompatibility of the host and guest models (caused by different grids, orography, physics, etc.). This can result in the imposed fields being out of balance, giving rise to noise, which may amplify and corrupt the forecast in the nested area. An essential preliminary step is to initialize both the host and guest models. As we saw in section 2, this may not be enough. The shock in the height error almost certainly points to a residual lack of 'smoothness' in the boundary fields. More rigorous mathematical guidance is needed. The issue of the need for smoothness when the boundary conditions are slightly inaccurate but contain useful information, has not, to my knowledge, been addressed in the context of transparent boundary conditions. In another context Browning and Kreiss (1982) made the point that if the boundary conditions 'are incorrect but smooth then we also showed that the ensuing solution was smooth. Unfortunately the error propagates at inertial/gravity wave speed so that the boundary data errors must be kept small'.

Because it is early days, some of the following remarks concerning the strengths and weaknesses of transparent boundary conditions are speculative. (a) Stability: in principle, we should be able to prove this for linear systems; see, for example, the appendix of Elvius and Sundström (1973). Lie (2001) derived stability for the mixed finite element discretization. It is not clear, however, that stability can be proven for the non-linear case. (b) Robustness has not been tested in a meteorological context. (c) These schemes are difficult to implement. The historical record speaks for itself. (d) Computational cost: this should be reasonable provided only lower order equations are needed. (e) Mass loss: mass loss is small except for the 'first order transparent b.c.' . (f) Over-specifying the boundary: we are now imposing the correct number of fields on the boundary, thus minimising the errors caused by faulty host model fields. (g) Geostrophic balance: we are no longer destroying quasi-geostrophic balance in a zone adjacent to the boundary. (h) Tuning parameters: there are none. If the scheme fails, it does so unambiguously. If it works, we do not have to re-tune any parameters if we change the grid size or time step. (i) Mathematically, its pedigree is excellent. (j) Residual reflections: waves for which $f/(\bar{c}k)$ is large, and also waves striking the boundary at acute angles will be strongly reflected. We argued in McDonald (2002b) that $f/(\bar{c}k)$ is small for most waves in a meteorological context. Also, consider a plane wave making an angle of 10° with the western boundary. After reflection it will later approach the northern boundary making an angle of 80°, guaranteeing almost zero reflection.

If the tested strategies prove inadequate, there are others on offer; see Tsynkov (1998). A particularly intriguing one which comes from electromagnetic theory, is known as the Perfectly Matched Layer technique; Berenger (1994). From a Meteorological perspective, it is interesting because it removes what we called *weakness (i)* of the relaxation scheme in our introduction. We meteorologists would think of it as a well-posed relaxation scheme. For an application to the linearized Euler equations see Hu (2001).

Lastly, if we take the lesson from our experiments that it is essential for the imposed boundaries to be as balanced as possible, and apply it to nested multi-level models, then initializing the host model fields using the guest forecast model may be advisable, particularly where the host model vertical levels, or orography, or physical parameterization schemes differ from those of the guest model. This may be expensive, but operationally it can be done off-line.

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