

3D-Var assimilation in transformed coordinates

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1 Introduction

The main reason for implementing a coordinate transformation in the assimilation is to get flow-dependent structure functions. The transformation used in this work is based on the geostrophic coordinates introduced by Hoskins[1]:

$$\begin{aligned} m &= x + v_g/f, \\ n &= y - u_g/f, \end{aligned} \tag{1}$$

where u_g, v_g are geostrophic winds and f is the Coriolis parameter. There is no transformation of the vertical coordinate.

This transformation has earlier been implemented (see Nordeng *et al.*[2]) in the norwegian NORANA assimilation system, which is a successive correction scheme working on pressure levels. Desroziers [3] has indicated how it could be implemented into a global 3D-Var system. In this work the method has been implemented in the HIRVDA 3D-Var assimilation system (version 4.4.0). The work is still “in progress”, but at least some preliminary results are presented.

2 Theory

The HIRVDA system uses the incremental formulation of 3D-Var, where (in standard notation, with $\delta x = x - x_b$) the cost function may be written:

$$J(\delta x) = \frac{1}{2}\delta x^T B^{-1}\delta x + \frac{1}{2}[y - h(x_b) - H\delta x]^T R^{-1}[y - h(x_b) - H\delta x]. \tag{2}$$

Here B and R are covariance matrices of background and observation errors, respectively. The observation error covariance matrix is assumed diagonal, but this is unrealistic for background error covariances. Therefore a new control vector χ is constructed, with the property that it has approximately statistically independent components, i.e., it has a diagonal (unit) error covariance matrix. This means that, with

$$\chi = T \delta x, \tag{3}$$

we have (with $d = y - h(x_b)$):

$$J(\chi) = \frac{1}{2}\chi^T \chi + \frac{1}{2}[d - HT^{-1}\chi]^T R^{-1}[d - HT^{-1}\chi]. \tag{4}$$

The transformation T is composed of a series of simpler transformations. Depending on the type of structure functions used, the transformation may be written:

$$T_{an} = P V L F C S^{-1} A \tag{5}$$

in case of analytical balance structure functions, and

$$T_{sb} = P V L S^{-1} B W F C \quad (6)$$

in case of statistical balance structure functions. The various transformations are explained in detail in [4] and [5]. Here we briefly note that S^{-1} and L are normalizations, F is Fourier transformation and P, V give a vertical decoupling via eigenvalue/eigenvector decompositions of vertical correlation matrices for the various effective spectral wave numbers. The A transformation is calculation of ageostrophic winds. The geostrophic part of the wind assimilation increments are controlled by the mass field (T, q, p_s) increments (thus the term analytical balance). For statistical balance, B is a transformation from unbalanced vorticity, divergence, temperature, surface pressure and humidity to their balanced counterparts based on regression equations. W is a transformation from vorticity, divergence back to u, v winds. Finally, C denotes the new coordinate transformation given by (1) (i.e., in standard 3D-Var $C = I$).

The main reason for the Fourier transformation is that homogeneity and isotropy of forecast errors in physical space implies statistical independence of spectral coefficients with different effective wave numbers. And the rationale behind the introduction of the coordinate transformation is that forecast errors are more likely to be homogeneous and isotropic in m, n space than in physical space. The coordinate change (1) has the property that it will give a stretching effect where the relative vorticity is positive and a contracting effect where it is negative. The vertical direction in transformed space is pointing towards colder air in a baroclinic flow, and will be tilted proportionally to the baroclinicity of the flow (for an explanation, see [2]).

3 Implementation

From (4) we see that only the inverse transformation T^{-1} and its adjoint $(T^{-1})^T$ is needed in the computation of the cost function and its gradient. This means that we need to compute C^{-1} instead of C . From e.g. (5), we have:

$$T_{an}^{-1} = A^{-1} S C^{-1} F^{-1} L^{-1} V^{-1} P^{-1}.$$

After the inverse (fast) Fourier transform F^{-1} , we now have assimilation increments on a regular grid in m, n space. We then need to determine values of assimilation increments on our original regular grid in x, y space, i.e.,

$$C_{an}^{-1} : \begin{bmatrix} \delta u_a \\ \delta v_a \\ \delta T \\ \delta q \\ \delta \ln p_s \end{bmatrix} (m, n) \longrightarrow \begin{bmatrix} \delta u_a \\ \delta v_a \\ \delta T \\ \delta q \\ \delta \ln p_s \end{bmatrix} (x, y).$$

Since it is straightforward to calculate m, n positions at the original x, y gridpoints using (1), C^{-1} only amounts to a (bilinear) interpolation of fields given on the regular m, n grid. The adjoint of that operation is also very simple. Observation operators are not affected, since these are applied after transformation back to physical space.

In the current implementation the transformation is constant, i.e., winds from the background field x_b are used. It is also possible to recompute the winds at each iteration. This

was tried in [2], but with no or negative impact. For simplicity, we have also used the same number of gridpoints in m, n space as in x, y space. Since points near the lateral boundaries of the original grid may well end up outside it in transformed space, the transformed grid area is in general larger than the original area. This results in a somewhat coarser average resolution than the original. But as noted above, in areas of positive relative vorticity, the resolution in transformed space will often be better than in physical space.

Theoretically, since an extension zone is used in conjunction with the Fourier transformation, values on m, n points outside of the original area could have been taken from values in the extension zone. But since fields are periodic on the extended area, observations near lateral boundaries would with this approach influence assimilation increments near the opposite lateral boundary, and this is probably not desirable. It seems more logical to stretch the grid.

There has been some concern that 3D-Var on a fine resolution grid covering an area of small horizontal extent might make it necessary to use a very large extension zone in order for structure functions not to influence fields across opposite lateral boundaries. Doing the assimilation in transformed coordinates seems to give another alternative. Since the transformation will in general stretch the area (this can be controlled by the user), influence across boundaries will be less in transformed space than in physical space. The price to pay in this case is that you get analysis increments on a coarser average resolution.

3.1 Parallelization

HIRVDA is of course a system capable of running on several processors. To achieve this the gridpoint area is divided (in the y direction) between processors. The same strategy is used in transformed space (subdivision in the n direction). With this strategy each processor does not hold all the m, n points it needs to interpolate back to its part of the x, y area. Thus some communication is needed in both C^{-1} and its adjoint. This has been implemented for both the SHMEM and MPI options in HIRVDA. It has also been verified that the transformation passes the inner product and gradient tests for correctness.

3.2 Structure function statistics

The HIRVDA system comes with a program to compute structure function statistics based on the NMC method. This method may be written tentatively as follows:

$$(W) F \Delta C A \begin{bmatrix} x_{0+48} \\ x_{24+24} \end{bmatrix} \longrightarrow \text{Cov}(k^*), (B)$$

The meaning of this is: The program takes ensembles of differences of +48 and +24 hour forecasts valid at the same time as input. Statistics for these error fields are computed (assuming homogeneity and isotropy), and a set of vertical covariance matrices for the various effective spectral wave numbers k^* (and a lot of regression coefficients in the statistical balance version) are produced as output. These matrices/coefficients are read in and used by the HIRVDA system to construct the P, V, L transformations mentioned previously. As indicated an option has been implemented for transformation to m, n space before generating these covariance matrices. Here we need to transform fields on a regular x, y grid to a regular m, n grid. This is not as simple as the inverse operation. In this first version a method based

on inverse interpolation (not requiring iterations) has been implemented, but this should probably be looked at more carefully.

4 Results and discussion

As a test the HIRVDA 3D-Var system was run with a single simulated temperature observation, both with and without coordinate transformation. Resulting assimilation increments are shown in Fig. 1, horizontally for the same layer as the observation, and for a vertical cross-section through the observation point.

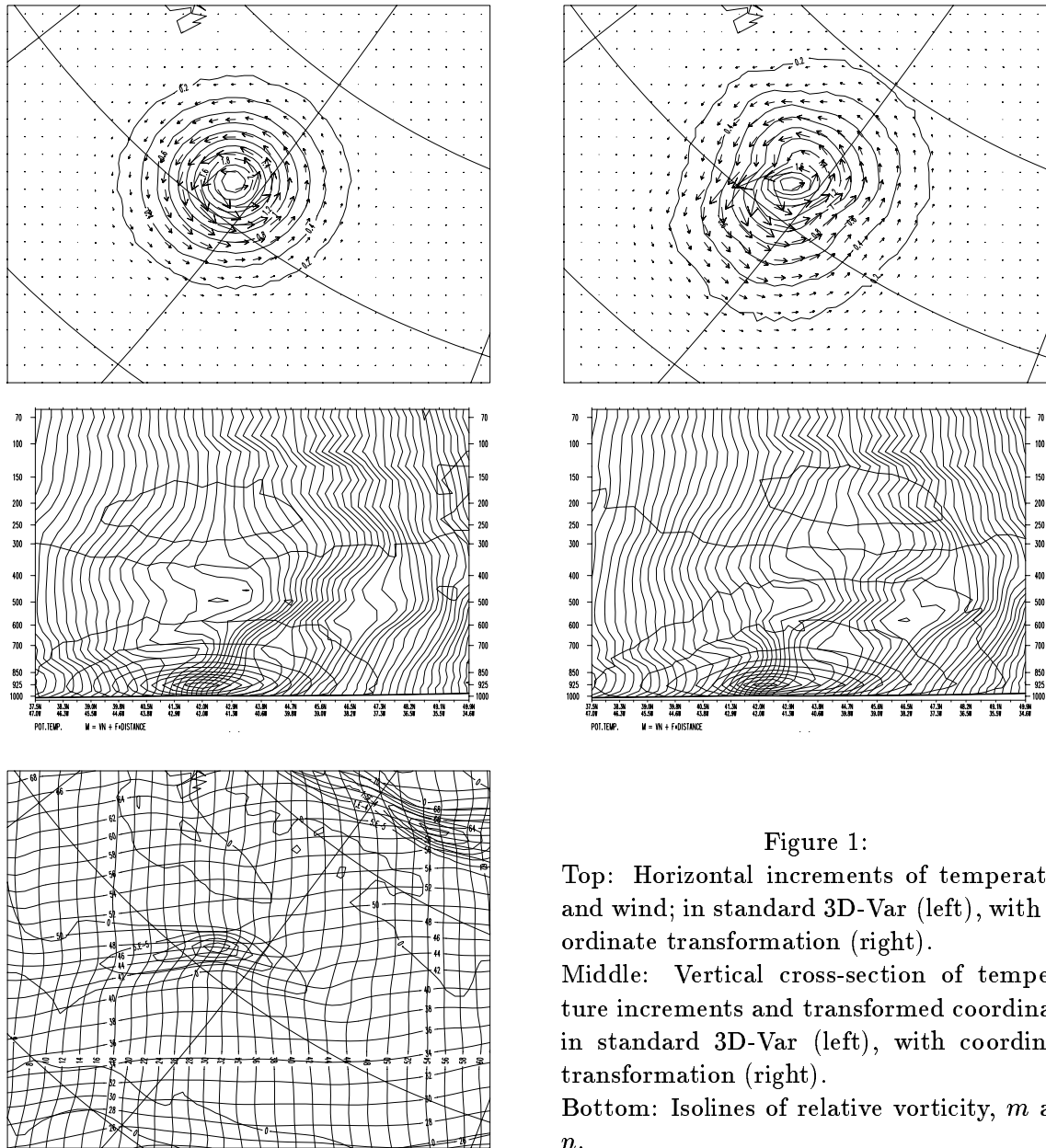


Figure 1:

Top: Horizontal increments of temperature and wind; in standard 3D-Var (left), with coordinate transformation (right).

Middle: Vertical cross-section of temperature increments and transformed coordinate; in standard 3D-Var (left), with coordinate transformation (right).

Bottom: Isolines of relative vorticity, m and n .

It can be verified that the transformation indeed produces flow-dependent, tilting assimilation

lation increments.

A parallel run with and without coordinate transformation has also been run, for a 3 weeks period in December 2000 on the DNMI operational 0.5° grid. Verification against EWGLAM stations for some selected parameters are shown in Fig. 2. The impact of the transformation seems to be neutral or slightly positive. This is however the first run performed, and more study is definitely needed before firm conclusions can be drawn.

There are a number of questions in the current implementation that needs to be sorted out by a closer study of the underlying theory, e.g.:

- Should the transformation be based on geostrophic wind or full wind? In [2] full wind was used, because geostrophic wind fields were not as smooth. Both options were implemented in this work. In the parallel run, both were tried, and geostrophic winds performed better, and are shown in Fig. 2.
- How to transform $\ln p_s$? So far the wind of the lowest model layer has been used, but there is no theoretical justification for it.
- When using statistical balance structure functions vorticity and divergence are calculated in spectral space, i.e., after the coordinate transformation is performed. Is this physically sound?

5 Conclusions

A momentum coordinate transformation is implemented and tested in HIRVDA 4.4.0. It gives flow-dependent assimilation increments that look reasonable, and the implementation seems to work technically.

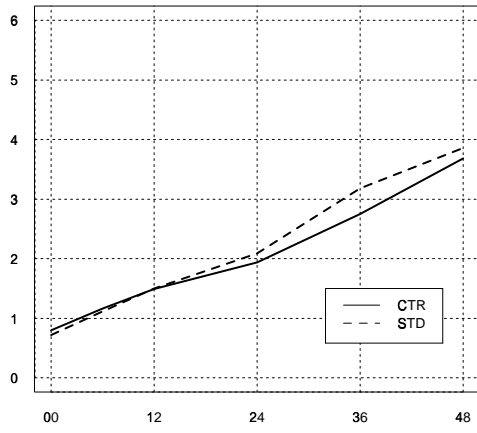
The first parallel test showed neutral or slightly positive impact when forecasts for a 3 weeks period in December, 2000 were verified against TEMP and SYNOP observations on EWGLAM stations.

There are several open theoretical questions that need to be studied more closely.

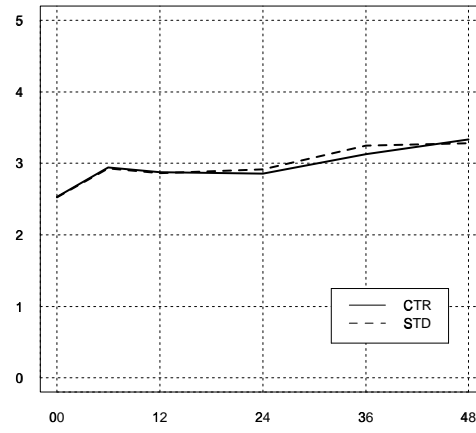
References

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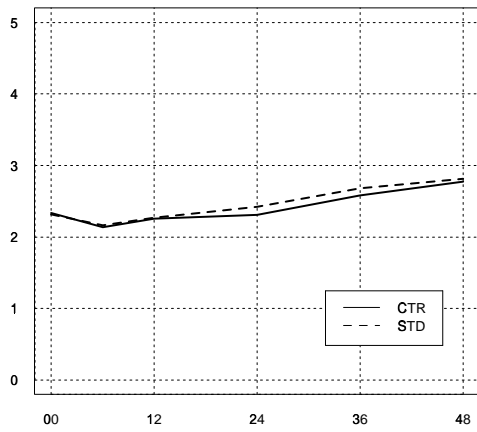
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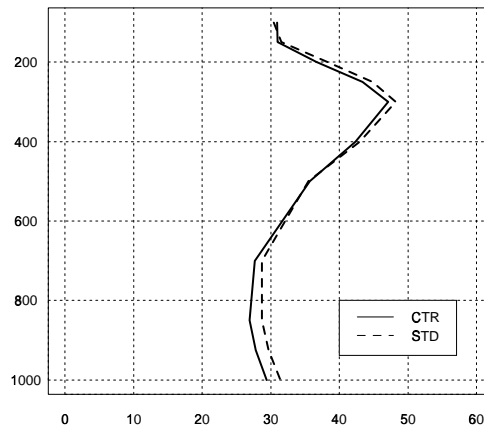
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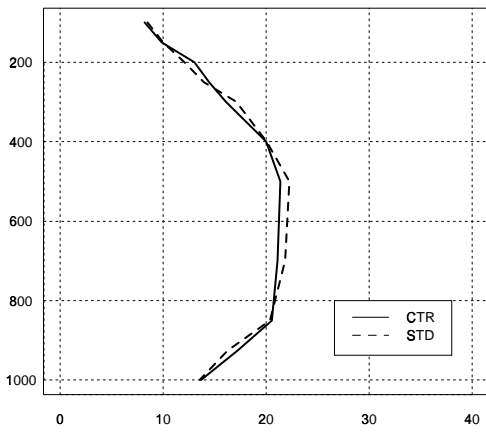
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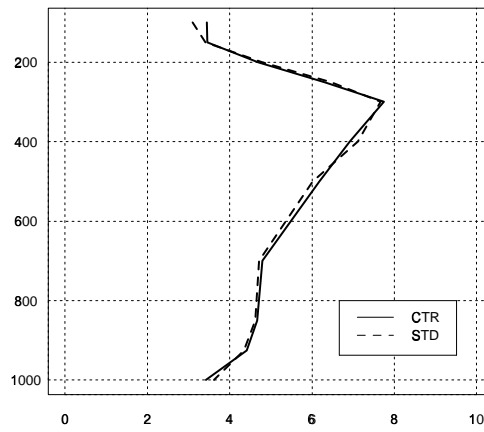


Figure 2: Verification scores, CTR = coordinate transformation, STD = standard 3D-Var