# CALCULATION TECHNIQUES OF NEAR-SURFACE TURBULENT FLUXES IN STABLE STRATIFICATION FOR NUMERICAL WEATHER PREDICTION MODELS

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## Summary

Practically oriented flux-calculation techniques based on correction functions to the neutral drag and heat/mass transfer coefficients are further developed. In the traditional formulation, the correction functions depend only on the bulk Richardson number. However, data from measurements of turbulent fluxes and mean profiles in stable stratification over different sites exhibit too strong variability in this type of dependencies. Indirect evidence from weather prediction and climate modelling also suggests that the traditional flux calculation technique is not sufficiently advanced. It is conceivable that other mechanisms besides the surface-layer stratification and, therefore, other arguments besides the bulk Richardson number must be considered. The proposed technique accounts for generally essential difference between the roughness lengths for momentum and scalars and includes a newly discovered effect of the static stability in the free atmosphere on the surface layer scaling. Recommended correction functions depend, besides bulk Richardson number, on one more stability parameter, involving the Brunt-Väisälä frequency in the free atmosphere, and on the roughness lengths.

## 1. Introduction

A starting point for practical calculation of turbulent fluxes in the surface layer is the Monin and Obukhov (MO) (1954) similarity theory. In stable stratification, it results in the familiar log-linear vertical profiles of the mean wind, $u$, potential temperature, $\theta$, and specific humidity, $q$. However, straightforward application of MO similarity theory is practically never used in numerical weather prediction (NWP) and climate models. It is not only infeasible to deal with a transcendental system of algebraic equations. Much more inconvenient is the fact that this system predicts non-zero turbulent fluxes only when the bulk Richardson number does not exceed some critical value (close to 0.3). At higher values of Ri, MO similarity theory suggests that the steady-state turbulence does not exist and all turbulent fluxes vanish, which means that the atmosphere becomes decoupled from the underlying surface.

The decoupling (although sometimes observed over limited areas) is unacceptable in NWPs for many reasons. In particular, once this has happened in a model run, the decoupling is often kept for a long time resulting in numerical instability. Moreover, there are clear physical grounds to exclude decoupling from coarse resolution models. Indeed, turbulent fluxes in NWPs represent area-average quantities, which practically never vanish due the contribution from unresolved (sub-grid scale) microcirculations.

Accordingly, flux-calculation techniques currently used in NWPs employ, instead of equations based on MO log-linear vertical profiles, modified formulations based on the drag coefficient, $C_D$, and the heat and mass transfer coefficients, $C_H$, and $C_M$,

$$C_D \equiv \frac{\tau_s}{u^2}, \quad C_H \equiv -\frac{F_{\theta s}}{u\Delta\theta}, \quad C_M \equiv -\frac{F_{qs}}{u\Delta q}. \tag{1}$$

Here, $\tau_s$ is the downward momentum flux per unit of mass; $F_{\theta s}$ and $F_{qs}$ are the near surface fluxes of potential temperature and humidity; $\Delta\theta$ and $\Delta q$ are the temperature and humidity increments. Suppose that $z_1$ is a fixed reference height over the surface within the surface layer, particularly, the height of the lower level in NWPs or the heigh of measurements, and $L = -u_* /(\beta F_{\theta s} + 0.61 g F_{qs})$ is the Monin-Obukhov length scale, $\beta = g/T$ is the buoyancy parameter. In neutral stratification [when $z_1 / L << 1$, so that log-linear profiles reduce to the well-founded logarithmic laws] the above coefficients become

$$C_{Dn} = \frac{k^2}{[\ln(z_1/z_{0u})]^2}, \quad C_{Hn} = \frac{kk_T}{\ln(z_1/z_{0u})\ln(z/z_{0T})}, \quad C_{Mn} = \frac{kk_q}{\ln(z_1/z_{0u})\ln(z/z_{0q})}. \tag{2}$$

The ratios $C_D / C_{Dn}$, $C_H / C_{Hn}$ and $C_M / C_{Mn}$ should decrease with strengthening of the static stability (with increasing $z_1 / L$). In the majority of NWPs this effect is taken into account, following Louis (1979) and Louis et al. (1982), through correction functions

$$f_D = C_D / C_{Dn}, \quad f_H = C_H / C_{Hn}, \quad f_M = C_M / C_{Mn}. \tag{3}$$

It is traditionally supposed that these functions depend only on the bulk Richardson number $Ri = (\beta\Delta\theta + 0.61 g\Delta q) z_1 / u^2$. Moreover, they are not rigorously derived from log-linear profiles. Such a derivation would inevitably lead to an unacceptable cut off the fluxes at large values of the bulk Richardson number Ri. Besides, it would result in correction functions dependent not only on Ri but also on the roughness lengths, $z_{0u}$, $z_{0T}$ and $z_{0q}$. Instead, the currently used correction functions $f_D$, $f_H$ and $f_M$ are approximated by positive single-valued functions of Ri, monotonically decreasing with increasing Ri and approaching zero as $\mathrm{Ri} \to \infty$. This type of parameterisation is to some extent supported by indirect evidence from weather prediction and climate modelling (e.g., DKRZ, 1993; Källen, 1996; Beljaars and Viterbo, 1998).

## 2. **Recent development in theory**

Zilitinkevich and Calanca (2000) distinguished between the familiar nocturnal stable boundary layers (SBLs), separated from the free atmosphere by near-neutral residual layers, and long-lived SBLs immediately adjoined to the stably stratified free atmosphere. They considered mean profiles in the surface layer within long-lived SBLs and disclosed essential dependence of the profiles on the average Brunt-Väisälä frequency $N$ in the adjacent layer of the free atmosphere, $N^2 = (\beta\partial\theta/\partial z + 0.61 g\partial q/\partial z)_{h<z<2h}$ where $h$ is the SBL height. This model was employed in the new surface-layer scheme within HIRLAM (Perov and Zilitinkevich, 2000). Zilitinkevich (2000) developed a theoretical model of the non-local turbulent transport, accounting for the internal-wave-induced interaction between the long-lived SBL and the free atmosphere. Zilitinkevich et al. (2000) evolved a practically orienterad flux-correction techniques based on correction functions to the neutral drag and heat/mass transfer coefficients. The model yields expressions for the drag correction function $f_D = C_D / C_{Dn}$ for a fixed height $z_1$,

$$f_D^{1/2} \equiv \left(\frac{C_D}{C_{Dn}}\right)^{1/2} \approx \frac{1 - a_u \mathrm{Fr}_{I0}}{\left(1 + \frac{C_u}{\ln(z_1/z_{0u})}\frac{z_1}{L}\right)}, \tag{4}$$

where $\mathrm{Fr}_{I0}$ is a version of the inverse Froude number based on external parameters (henceforth referred to as the external Froude number), $\mathrm{Fr}_{I0} = Nz_1/u$

Accordingly, the virtual-potential-temperature transfer (VPTT) correction function $f_{\theta v}$ for a fixed height $z_1$ become

$$f_{\theta v} \equiv \frac{C_{\theta v}}{C_{\theta vn}} = \frac{1 - a_\theta \mathrm{Fr}_I \frac{\mathrm{Fr}_{I0}^2}{\mathrm{Ri}}}{\left(1 + \frac{C_\theta}{\ln(z_1/z_{0T})}\frac{z_1}{L}\right)}, \tag{5}$$

where the neutral VPTT coefficient $C_{\theta vn}$ is given by $C_{\theta vn} = k_T / \ln(z_1/z_{0T})$. In Eqs. (4), (5), the Monin-Obukhov length $L$ and, therefore, the ratio $\varsigma \equiv z_1/L$ is an internal parameter dependent on turbulent fluxes. External parameters, immediately available within NWPs, are the bulk Richardson number Ri and the external inverse Froude number $\mathrm{Fr}_{I0}$, and two parameters dependent of the roughness lengths $\lambda_u = \ln z_1/z_{0u}$, $\lambda_T = \ln z_1/z_{0T}$. To make the above equations practically useful, the unknown $\varsigma$ should be expressed through the external parameters.

## 3. Surface layers within nocturnal SBLs

As mentioned above, nocturnal SBLs are separated from the free atmosphere by near-neutral residual layers, where the static stability is weaker than within SBLs. As a result, internal waves do not radiate upward from the upper boundary of this type of SBLs. This affords that the turbulent transport becomes basically local (see Z). In this regime, the surface layer turbulence is well described by the Monin-Obukhov similarity theory, provided that the static stability is not too strong. Remember that the formulation given in Section 2, namely Eqs. (4), (5), reduces to the Monin-Obukhov-theory based formulation taking $N$=0 and $\mathrm{Fr}_{I0}$=0. Then, drag and the VPTT correction functions become

$$f_D^{1/2} \equiv \left(\frac{C_D}{C_{Dn}}\right)^{1/2} = \frac{1}{1+\frac{C_u}{\lambda_u}\varsigma} \approx \frac{1}{1+\frac{\mathrm{Ri}}{\mathrm{Ri}_1}\left(1+\frac{\mathrm{Ri}}{\mathrm{Ri}_1}\right)} \tag{6}$$

$$f_{\theta v} \equiv \frac{C_{\theta v}}{C_{\theta vn}} = \frac{1}{1+\frac{C_\theta}{\lambda_\theta}\varsigma} \approx \frac{1}{1+\frac{C_\theta \lambda_u}{C_u \lambda_T}\frac{\mathrm{Ri}}{\mathrm{Ri}_1}\left(1+\frac{\mathrm{Ri}}{\mathrm{Ri}_1}\right)} \tag{7}$$

## 4. Surface layers within long-lived SBLs

The model under consideration is applicable to strongly stable stratification (see Z), that is to large values of $\varsigma = z_1 / L$ and Ri (as distinct from the traditional model presented in Section 3). Drag correction function in this case becomes

$$f_D^{1/2} \equiv \left(\frac{C_D}{C_{Dn}}\right)^{1/2} = \frac{1 - a_u \mathrm{Fr}_{I0}}{\left(1+\frac{C_u}{\lambda_u}\frac{A_1 A_2}{\mathrm{Ri}_2 - \mathrm{Ri}}\right)}. \tag{8}$$

Here, $\mathrm{Ri}_2$ is the critical bulk Richardson number ($\mathrm{Ri}_2 \sim 5$ at very strong free flow stability), $A_1$ and $A_2$ are variable coefficients dependent on external parameters only. Considering the VPTT coefficient, remember once again that the regime under consideration relates to Richardson numbers Ri close to $\mathrm{Ri}_2$.

$$f_{\theta v} \equiv \frac{C_{\theta v}}{C_{\theta vn}} = \frac{A_2}{\left(1+\frac{C_\theta}{\lambda_T}\frac{A_1 A_2}{\mathrm{Ri}_2 - \mathrm{Ri}}\right)}. \tag{9}$$

Eqs. (6) and (7) are relevant to regimes with very small values of $\mathrm{Fr}_{I0}$ and not too large values of Ri. On the contrary, Eqs (8) and (9) are absolutely irrelevant in these regimes. They acquire physical sense only when both Ri and $\mathrm{Fr}_{I0}$ become large. For practical use within NWPs, these two alternative formulations should be combined. A reasonable interpolation in terms of the drag and the heat/mass transfer correction functions read

$$f_D = \max\left\{\left[f_D \text{ after Eq. (6)}\right],\ \left[f_D \text{ after Eq. (8)}\right]\Phi\left(\mathrm{Fr}_{I0}\right)\right\}, \tag{10}$$

$$f_H = f_M = \max\left\{\left[f_D^{1/2} f_{\theta v} \text{ after Eqs.} (6, 7)\right], \left[f_D^{1/2} f_{\theta} \text{ after Eqs.} (8,9)\right]\Phi(\mathrm{Fr}_{I0})\right\}. \qquad (11)$$

Here, $\Phi$ is a weight function, $\Phi = \mathrm{Fr}_{I0}{}^2/(\mathrm{C}+\mathrm{Fr}_{I0}{}^2)$ and $C$ is a dimensionless coefficient, which should be chosen by fitting Eqs. (10)-(11) to experimental data on the correction functions. Its currently recommended value is $C$=0.05. The role of the weight function is to rule out the formulation, given by Eqs. (8) and (9), when it becomes inapplicable (that is when $\mathrm{Fr}_{I0}$ diminishes).

Notice that Eqs. (8) and (9) cut off the turbulent fluxes at $\mathrm{Ri} > \mathrm{Ri}_2$. Nevertheless, the recommended interpolation, Eqs. (10)-(11), prevents their total annihilation due to the approximation deliberately adopted in (6) and (7).

To a reasonable approximation, it is sufficient to approximate $N$ by its mean value within a reasonably deep layer placed immediately above the upper boundary of typical SBLs, e.g., 250 m < $z$ <750 m.

## 8. Comparison with experimental data

In this section the recommended correction functions, Eqs. (10)-(11), are compared with two sets of data, representing essentially different measurement sites, Halley in Antarctica and Sodankyla in Arctic Finland.

Data at Halley Research Station were collected during 1997. Halley is situated on a very flat and uniform ice sheet. Here, the measured roughness parameter for wind was very small, $z_{0u} = 5.6\times10^{-5}\,m$, which is typical of smooth snow surfaces; whereas the measured scalar roughness $z_{0T}$ was essentially variable and unusually large, much larger than $z_{0u}$ (King and Anderson, 1994). The measured correction functions and bulk Richardson numbers were deduced from hourly mean data on the surface fluxes, the snow-surface temperature $\theta_s$, the air temperature $\theta(z_1)$ and the wind speed $u(z_1)$ at the heights $z_1$ equal to 3.3 *m* – 4.8 *m* depending on month.

The conditions in the lower troposphere were monitored once a day with radiosondes. The Brunt-Väisälä frequency $N$ in the free flow was calculated from the 200 *m* – 400 *m* potential temperature difference (the SBL depth at this station was generally less then 150 m). Its typical value was $N = 2\times10^{-2}\,s^{-1}$. Given the height of measurements $z_1$~4 *m* and the wind speed $u$~4 $m\ s^{-1}$, the external inverse Froude number $\mathrm{Fr}_{I0}$, Eq. (17), becomes rather small ($\sim 2\times10^{-2}$). It is not surprising, therefore, that the effect of the free-flow stability on the correction functions was practically not seen in these data. Clearly, with greater values of $z_1$, say with $z_1 = 35\,m$ – as in the operational HIRLAM, this effect would become strongly pronounced.

Sodankyla is a site about 100 km to the North from the Arctic Circle, Finland. Data analysed in this paper were obtained during January 2000 from measurements on a 48 *m* height tower surrounded by a high (15 – 20 *m*) and very sparse forest. The turbulent fluxes and the mean wind were measured at 4 different levels and the temperature was measured at 7 levels. The two lower levels (3 *m* and 8 *m*) were within the forest and the other three, above the forest. The measured snow-surface temperature, the above turbulent fluxes and the mean wind and temperature at $z_1 = 32\,m$ were used. This height is close to the height of the first level in the weather prediction model HIRLAM. The measured roughness length for momentum at Sodankyla was $z_{0u} = 0.8$ *m* (much larger than at Halley). The measured roughness for temperature $z_{0T}$ was rather small and strongly variable. The correction functions and bulk Richardson numbers were deduced in the same way as at Halley from hourly mean data.

Figure 1 (a and b) presents the scatter plots of the drag and the heat/mass-transfer correction functions versus the bulk Richardson numbers for Halley. The crosses show data from measurements. The solid lines are calculated after Eq. (10) – in Figure 1a and Eq. (11) – in Figure 1b. To reflect the variability in the scalar roughness lengths, triples of curves are presented – for the following external parameters:

$$\mathrm{Fr}_{I0} = 2\times10^{-2}, \quad \lambda_u = 11.2, \quad \lambda_T = \begin{cases} 6.6 & \text{for } z_{0T} = 10^2 z_{0u} \text{ (upper curve)} \\ 4.3 & \text{for } z_{0T} = 10^3 z_{0u} \text{ (medium curve)} \\ 2.0 & \text{for } z_{0T} = 10^4 z_{0u} \text{ (lover curve).} \end{cases} \qquad (12)$$

In this figure, the effect of the free-flow stability is not seen because of too the low reference height $z_1$.

The similar correction functions were calculated for Sodankyla (not shown). They noticeably differ from those in Figure 1. This is only natural. Here, the measurement height $z_1$=32 *m* is much higher than $z_1$= 3.3 – 4.8 *m* at Halley. This means that the inverse Froude number and the bulk Richardson number have different sense in the two figures. Even more important is the fact that the roughness lengths at the considered two sites differ very strongly.

In Figure 2, data from Sodankyla are employed to illustrate the effect of the free-flow stability on the correction functions. The latter are plotted for the fixed values of $\lambda_u$=3.7 and $\lambda_T$=6.0, and three alternative values of $\mathrm{Fr}_{I0}$, namely, $\mathrm{Fr}_{I0}$=0 (nocturnal SBL), $\mathrm{Fr}_{I0}$=0.2 (typical of Sodankyla), and $\mathrm{Fr}_{I0}$=0.3 (the maximum observed value corresponding to clearly defined long-lived SBL). Figure 2 suggests that the “nocturnal SBL” curve and the “long-lived SBL” curve represent reasonable frames for data points at large Ri.

As seen in all figures, the recommended correction functions are basically consistent with available experimental data. Such a consistency cannot be achieved with any single-valued dependence of Ri. Thus the very concept of the single-valued correction functions is an oversimplification.

## 9. Conclusions

The proposed turbulent flux calculation technique incorporates in the practical context a recently developed theoretical model of the non-local turbulent transport accounting for the effect of the internal-wave interaction between the surface layer and the free atmosphere. The key role in this distant link is played by the Brunt-Väisälä frequency *N* in the layer immediately above the SBL. As a result the traditional surface-layer scaling and the flux calculation technique are extended including *N* as an additional governing parameter.

The new technique is presented in the conventional format, through the neutral drag and heat/mass transfer coefficients and the stability-dependent correction functions. However, the latter are no longer considered as single-valued functions of the bulk Richardson number. They depend also on $N$ and on the ratio of the roughness lengths for momentum and scalars.

The new technique is validated through comparison with data from measurements over essentially different land surfaces, such as a very smooth bare snow and a very rough boreal forest. It predicts essentially higher level of turbulence than formerly recognised at large values of the bulk Richardson numbers, in good correspondence with indirect evidence from weather prediction and climate modelling.

Although more comparison with experimental data is needed, the proposed technique deserves fast implementation in NWPs. First successful attempt to employ the new surface-layer scheme within 1-D HIRLAM has already been made by Perov and Zilitinkevich (2000).

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## References

Beljaars, A. C. M., and Viterbo, P. 1998 Role of the boundary layer in a numerical weather prediction model. *Proceedings of the Colloquium "Clear and Cloudy Boundary Layers"*, 26-29 August 1997, 287-304.

DKRZ (Ed.) 1993 *The ECHSAM3 Atmospheric General Circulation Model*. Deutsches Klimarechenzentrum (DKRZ) Technical Report No. 6, 189 pp. (Available at Max Planck Institute for Meteorology, Bundesstrasse 55, DE-20146 Hamburg, Germany.)

Källen, E. (Ed.) 1996 *HIRLAM Documentation Manual – System 2.5*. (Available at Swedish Meteorological and Hydrological Institute, Swedish Meteorological and Hydrological Institute (SMHI), S-601 76 Norrköping, Sweden.)

King, J. C., and Anderson, P. S. 1994 Heat and water vapour fluxes and scalar roughness lengths over an Antarctic ice shelf. *Boundary-Layer Meteorol*., **69**, 101-121

Louis J. F., 1979: A parametric model of vertical fluxes in the atmosphere. *Boundary-Layer Meteorol.*, **17**, 187-202.

Louis J. F., Tiedtke, M., and Gelen, J. F. 1982: A short history of the PBL parameterization at ECMWF. *Proceedings of Workshop on Boundary Layer Parameterization*, ECMWF, Reading, 59-79.

Monin A. S., and Obukhov, A. M. 1954 Basic laws of turbulence mixing in the surface layer of the atmosphere. *Trudy Geofiz. Inst. AN SSSR*, **24** (151), 163-187.

Perov V. and Zilitinkevich S. 2000 Application of an extended similarity theory for the stably stratified atmospheric surface layer to HIRLAM. *HIRLAM Newsletter* **35**, 137-142.

Zilitinkevich, S.S., Perov V.L., and King J.C. 2000 Near-surface turbulent fluxes in stable stratification: calculation techniques for use in general circulation models. Submitted to *Quart. J. Roy. Meteor. Soc.*

Zilitinkevich, S. S. 2000 Third-order transport due to internal waves and non-local turbulence in the stably stratified surface layer. Submitted to *Quart. J. Roy. Meteorol. Soc.*

Zilitinkevich, S., and Calanca, P. 2000 An extended similarity-theory for the stably stratified atmospheric surface layer. *Quart. J. Roy. Meteorol. Soc.*, **126**, 1913-1923.

## Figure captions

**Figure 1.** The correction functions (a) to the drag coefficient, $f_D$, and (b) to the heat and mass transfer coefficients, $f_H = f_M$, versus the surface-layer bulk Richardson number Ri. Crosses are data from measurements at Halley, Antarctica. The reference height is $z_1$ = 3.3 – 4.8 *m*. The curves are calculated after Eqs. (10), (11) for three values of the ratio $z_{0u} / z_{0T}$, namely, $10^{-2}$, $10^{-3}$ and $10^{-4}$ – for the upper, medium and lower curve, respectively.

**Figure 2.** The effect of the free-flow stability on the correction functions by the example of Sodankyla. The curves are calculated after Eqs. (10), (11) for $\lambda_u$=3.7, $\lambda_T$=6.0, and three values of the external inverse Froude number, $\mathrm{Fr}_{I0} = 0$, $\mathrm{Fr}_{I0} = 0.2$, and $\mathrm{Fr}_{I0} = 0.3$, representing different regimes from the nocturnal SBL to the clearly defined long-lived SBL.

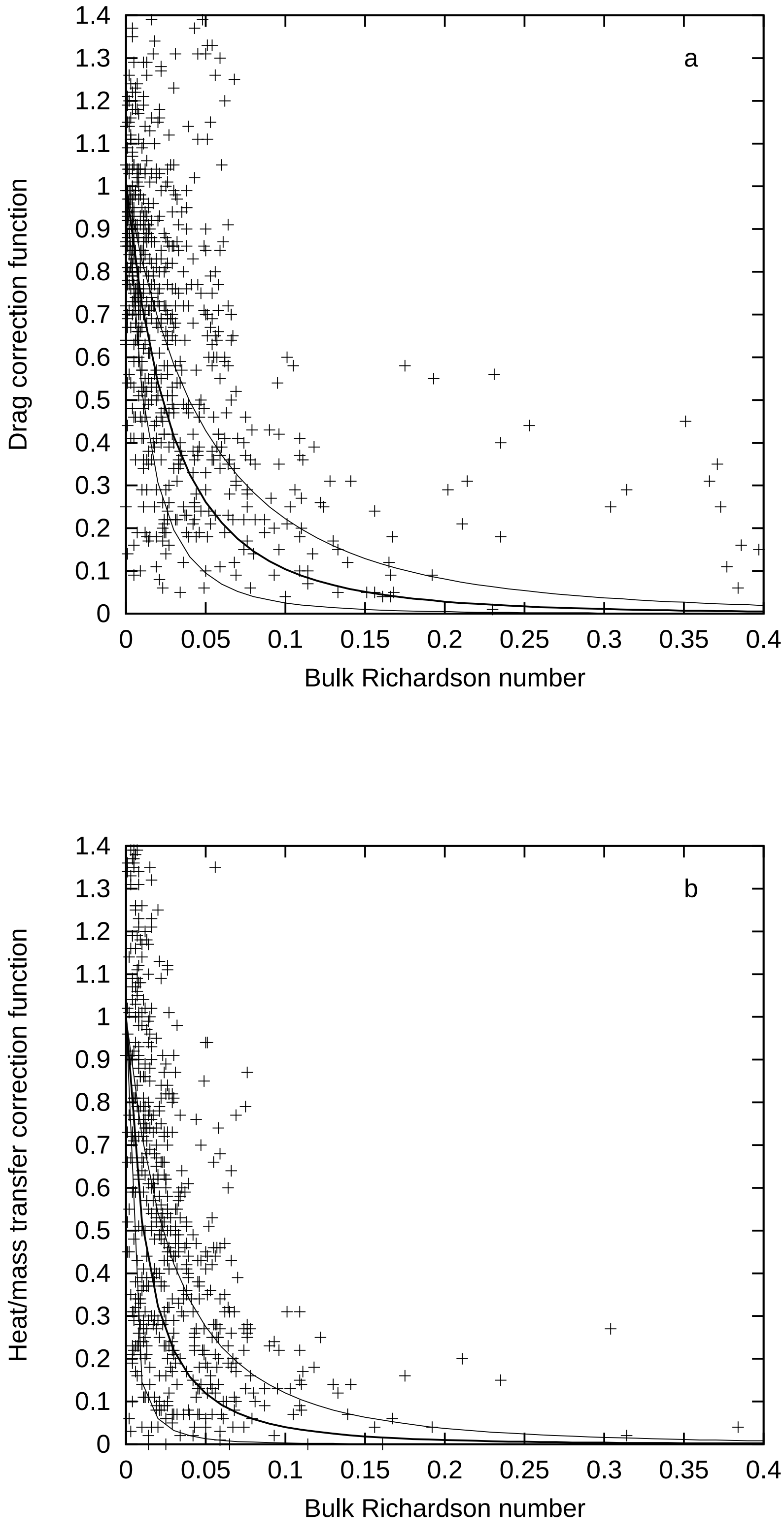
a
Drag correction function
Bulk Richardson number
b
Heat/mass transfer correction function
Bulk Richardson number


Figure 1.

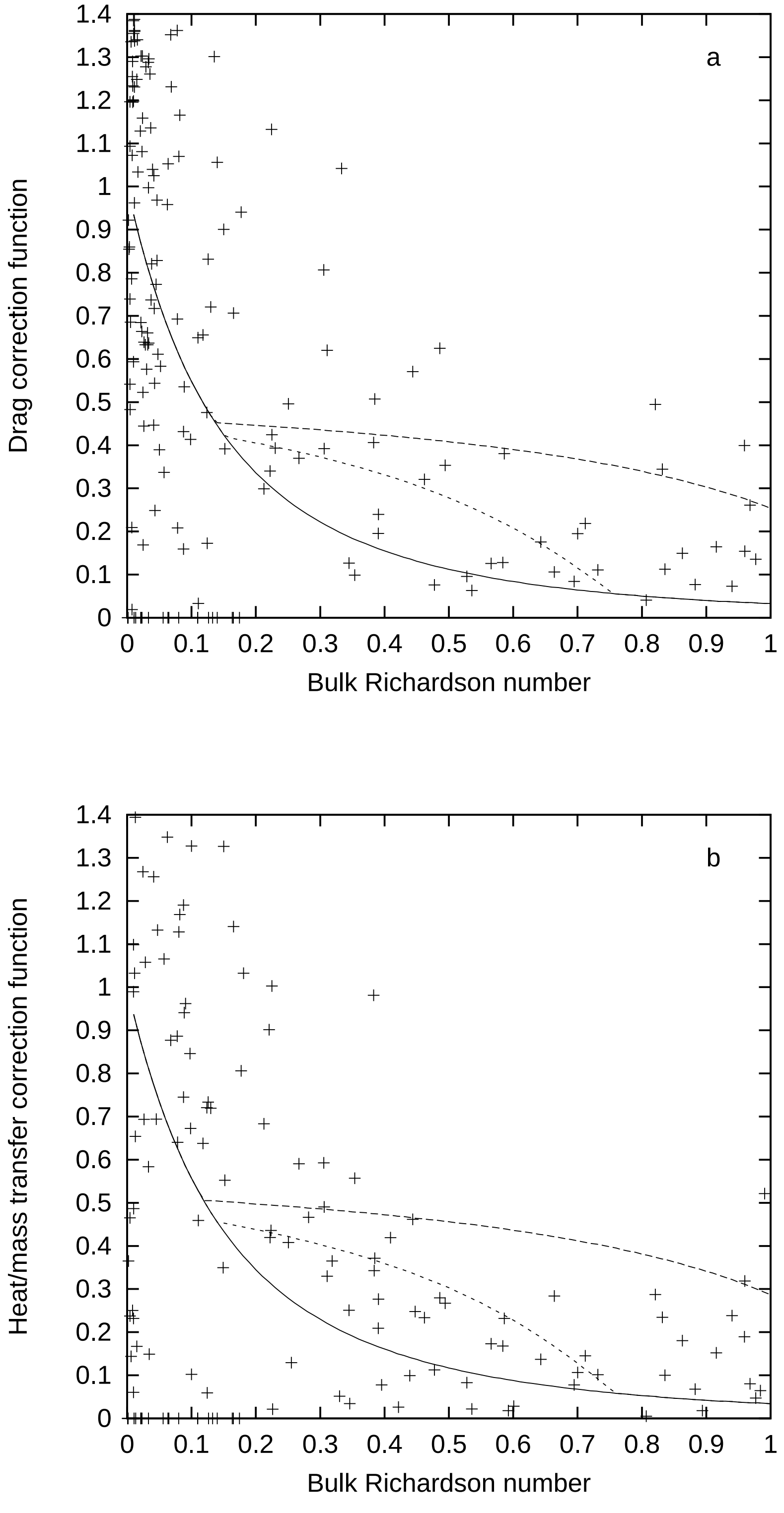
a
Drag correction function
Bulk Richardson number
b
Heat/mass transfer correction function
Bulk Richardson number
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4

Figure 2.