HIRLAM Dynamics status and plans

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1 Introduction

This article summarizes the presentation given at the All Staff Meeting in Oslo about the status and plans of the dynamics project in HIRLAM at the time of the meeting.

2 Research subjects

- Vertical Finite Element discretization of the HARMONIE non-hydrostatic version
- Use of non-constant linearized map-factor for large areas
- Frequent update of lateral boundaries
- Transparent boundary conditions
- Dynamics-physics interface
- Conservation in semi-Lagrangian advection
- Semi-elastic model as an alternative to fully non-hydrostatic model

3 Description of status and plans

1.1 Vertical Finite-element discretization of the non-hydrostatic version of the forecast model

The vertical coordinate used in the vertical discretization of the HARMONIE model is the terrain-following hydrostatic-pressure coordinate proposed by Laprise (1992). In this coordinate system, the set of forecast equations is as follows:

\[
\begin{align*}
\frac{d}{dt} \mathbf{V} + \frac{RT}{p} \nabla_q p + \left( \frac{1}{m} \frac{\partial p}{\partial \eta} \right) \nabla_q \phi &= F \\
\frac{dw}{dt} + g \left( 1 - \frac{1}{m} \frac{\partial p}{\partial \eta} \right) &= F_z \\
\frac{dT}{dt} - \frac{RT}{pC_p} \frac{dp}{dt} &= \frac{Q}{C_p} \\
\frac{1}{p} \frac{dp}{dt} + \frac{C_p}{C_v} D_3 &= \frac{Q}{C_v T} \\
\frac{d\phi}{dt} - \omega g &= 0 \\
\frac{dm}{dt} + mD + \frac{\partial \hat{\eta}}{\partial \eta} &= 0
\end{align*}
\]

(1)

Here, \( \mathbf{V} \) is the horizontal component of the wind vector, \( w \) its vertical component, \( T \) the temperature, \( p \) the pressure, \( \phi \) the geopotential and \( m \) the vertical derivative of the hydrostatic pressure \( \pi \).

\[
m \equiv \frac{\partial \pi}{\partial \eta}
\]

(2)

D3 is the three-dimensional divergence, defined as:
The geopotential is related to the other variables through the relationship

\[
\frac{\partial \phi}{\partial \eta} = -m \frac{RT}{p} \Rightarrow T = -\frac{p}{mR} \frac{\partial \phi}{\partial \eta}
\]

(4)

The right-hand-side of the equations include the physical parameterizations and the horizontal diffusion.

Linearizing the non-advection terms of the forecast equation around a constant reference temperature \( T^* \) and a constant reference surface hydrostatic pressure and \( \pi'_s \) applying the semi-Lagrangian discretization to the advection terms, we obtain the linear system:

\[
D^* + \frac{\Delta t}{2} \nabla^2 \phi^* + \frac{RT^*}{\pi^*} \frac{\Delta t}{2} \nabla^2 p^* = R_D
\]

\[
w^* + \frac{\Delta t}{2} \frac{g}{m} m^* - \frac{\Delta t}{2} \frac{g}{m} \frac{\partial}{\partial \eta} p^* = R_w
\]

\[
p^* + \frac{C_p}{C_v} \frac{\Delta t}{2} \pi^* D^* - \frac{C_p}{C_v} \frac{\Delta t}{2} \frac{RT^*}{m} \frac{\partial w^*}{\partial \eta} = R_p
\]

\[
\phi^* - \frac{\Delta t}{2} w^* = R_\phi
\]

\[
m^* + \frac{\Delta t}{2} m^* D^* = R_m
\]

(5)

Which is the basis for applying the semi-implicit method to the solution of the forecast equations. The terms \( R \) on the right-hand-side include the non-linear terms of the equations.

Notice that there is no Temperature equation in this set. The temperature will be computed as a diagnostic quantity using equation (4).

Eliminating unknowns in the set of equations (5) we arrive at a single equation for \( w^* \):

\[
\left \{- \frac{1}{c_s^2} + \frac{(\Delta t)^2}{2} \left[ \nabla^2 + \frac{1}{H^2} \left( \frac{\pi^*}{m^*} \frac{\partial}{\partial \eta} \left( \frac{\pi^*}{m^*} \frac{\partial}{\partial \eta} \right) \right] \right \} \left( \frac{\Delta t}{2} \right)^4 N^2 \nabla^2 \right w^* = R_c
\]

(6)

Here

\[
c_s^2 \equiv \frac{C_p}{C_v} \frac{RT^*}{g}; \quad H \equiv \frac{RT^*}{g}; \quad N^2 \equiv \frac{g^2}{C_p T^*}
\]

In the elimination process, no constraint similar to “constraint C1” or “constraint C2” of Bubnová et al (1995) is found when using the discretized form of the continuous operator \( \partial / \partial \eta \).
Notice that in the set of equations chosen, the only vertical operator is the derivative operator. This operator will be computed using a set of basis functions in the domain $0 < \eta < 1$ similar to the cubic splines used by Untch & Hortal (2004) for the hydrostatic set of equations where the only vertical operator was the integral operator.

The solution procedure will then be analogous to the procedure used in the hydrostatic model. The r.h.s. of the Helmholtz equation (6) will be projected onto the eigenvectors of the “vertical laplacian” operator

$$L \equiv \frac{\pi^*}{m^*} \frac{\partial}{\partial \eta} \left( \frac{\pi^*}{m^*} \frac{\partial}{\partial \eta} \right) + \frac{\pi^*}{m^*} \frac{\partial}{\partial \eta}$$

In order to decouple in the vertical the set of equations (6). The solution of each component will be stable if the corresponding eigenvalue of operator $L$ is real and negative. Once the value of $w^*$ is found, the others will be computed by substitution.

Research is at the moment going on in order to find the exact set of basis functions which will provide a discretized vertical laplacian operator in which all the eigenvalues are real and negative.

1.2 Use of non-constant linearized map factor for large areas

Yessad & Bénard (1996) describe an instability in the ARPEGE stretched model when using a constant value of the map factor in the linearized equations of the semi-implicit method. They propose a solution using, in this set of equations, a map factor which is a combination of Legendre polynomials of orders zero and one only. The price to pay is that the set of Helmholtz equations is not completely decoupled in the horizontal and the solution procedure includes solving a pentadiagonal set.

In the limited-area version of the forecast model (the HARMONIE model), a similar problem appears as the model is solved on a projection where the map factor is not constant. A solution similar to the one applied by Yessad & Bénard in the global model (Voitus 2004) demands that the map factor can be expressed as a linear combination of only a few low-order Fourier components.

This is most easily achieved on a Mercator projection, where the map factor depends only on the $y$ direction and has an analytical expression. The Mercator projection is available in the HARMONIE model thanks to the work of D. Gril.

A study has been made on the fit of the map factor on a Mercator projection using different numbers of Fourier components in the $y$ direction. An example is shown in Figure 1 using three Fourier components.

The stability of the non-linear model with non-constant map-factor in the linearized version will be
studied, for the two-time-level semi-Lagrangian scheme, using a method similar to the one used by Simmons & Temperton (1997) to determine the reference temperature and reference pressure in the linearization of the hydrostatic model.

1.3 Frequent update of the lateral boundary conditions
Given the difficulties of developing proper transparent boundary conditions for non-linear non-hydrostatic models (see next paragraph), one way to improve the present situation, within the philosophy of “overspecify and relax” is to update the lateral boundary conditions coming from the host model as frequently as possible. In particular it would be best to update at every time step of the host model and try to adapt as best as possible these boundary conditions to the “balance” of the high-resolution guest model.

The adaptation of the boundary conditions to the guest model can be tried by using the orography of the high-resolution model in the relaxation area, by using weak coupling in the lowest model levels, by tuning the relaxation coefficients or by other means to be investigated.

1.4 Transparent lateral boundary conditions
The proper strategy to treat the lateral boundaries accurately has been defined by Engquist & Majda (1997). This strategy has been shown to work very well for linear systems in grid-point models and for one-dimensional spectral models. It needs to be adapted to fully non-linear systems and to the two-dimensional spectral representation used in the HARMONIE model. One of the possible strategies for applying transparent boundary conditions to spectral models is to “externalize” the boundary conditions (see Termonia & Voitus (2008)).

A road map has been agreed between HIRLAM and ALADIN for this adaptation, which needs several intermediate steps, to make sure the chosen direction is the proper one.

1.5 Dynamics-Physics interface
The present interface between the physics and the dynamics is first-order accurate in time (and therefore in a semi-Lagrangian environment, also first order accurate in space) and allows only the same space resolution in both parts of the forecast model.

The inclusion in the Danish HIRLAM team of a new member dedicated to the developments in dynamics, has allowed to start the work aimed to improve on these two limitations.

Running the physics at a resolution different from the dynamics has been suggested by several authors in the past and was implemented for the ECMWF physics in the global IFS model. Following these lines, the same possibility is being included in the interface with all the physics packages available in the HARMONIE model.

Later on, and in close collaboration with the ALADIN group, the interface will be extended to allow a second-order accurate treatment of the physical parameterizations.

1.6 Conservation in semi-Lagrangian advection
The semi-Lagrangian advection scheme is not designed for conservation of the advected field, but for accuracy and stability when using long time-steps in the forecast model.

Several schemes exist which conserve mass, based on the finite-volume approach. The drawback of these schemes is that they are subject to very restrictive CFL conditions which make the running of the model much more expensive.
Other schemes (i.e. the cell-integrated semi-Lagrangian scheme) are very complicated and expensive in terms of computer time and finally others apply a-posteriori fixes for conservation which decrease the accuracy of the advection in favour of the formal conservation.

Recently Kaas (2008) has proposed a locally mass-conserving scheme based on the refinement of the positions of the departure points of the semi-Lagrangian trajectories in such a way that the mass is conserved. This scheme probably does not decrease the accuracy of the semi-Lagrangian advection, could be easy to implement and cheap to run and it conserves mass. This scheme will be tried for the continuity equation when a full three-dimensional version is available and has been tested in idealized environments.

The conservation of the total mass of dry air is a necessary but not sufficient condition for the conservation of the total amount of a tracer or a chemical species, because the semi-Lagrangian procedure is applied to the mass ratio of the tracer.

The different options available in the semi-Lagrangian interpolation of the HARMONIE model will be compared from the point of view of conservation, when applied to different tracer distributions in which the only tendencies come from the advection (no chemistry for ozone, no wet physics for water vapour). Combinations of them or new interpolation procedures will be included in order to improve the conservation without compromising the accuracy.

1.7 Semi-elastic model as an alternative to fully non-hydrostatic model
The semi-elastic scheme proposed by Rõõm & Männik (2006) filters, in principle, the acoustic waves present in a fully non-hydrostatic model and therefore eliminates a possible source of instability in the forecasts. Nevertheless, recent tests have had stability problems which might need the introduction of a predictor-corrector scheme.

If this model proves to be more efficient in computational terms than the fully non-hydrostatic HARMONIE, a series of real cases will be run and compared in quality with reference runs, using the verification tools developed.

References


Voitus F (2004): Liens entre la geometrie horizontale et le schema semi-implicite dans les modeles ARPEGE, ARPEGE/ALADIN et ARPEGE/ALADIN-NH. *Rapport de stage de fin d'études no 951, Ecole Nationale de la Meteorologie, France*. 53 pp