Plan for research in lateral boundary conditions

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1 Boundary strategy for large area models (hydrostatic)

The standard lateral boundary treatment uses the fields from a global model to update the boundaries, allied with a zone adjacent to the boundary in which the guest fields are relaxed toward the host global fields. This is a source of error for two reasons. (a) All the fields are imposed (over-specification) and (b) the fields are only updated every 3 (sometimes 6) hours.

These errors can propagate from the boundary into the forecast area at essentially two speeds: (i) ‘slow’ (advective-Rossby waves) and ‘fast’ (gravity-inertia waves). The ‘slow errors’ are taken care of by situating the lateral boundaries far enough away from the region of interest so that they cannot travel there during the duration of the forecast. The ‘fast errors’ are eliminated by having a damping zone adjacent to the the boundary. In that zone the damping mechanism is tuned to eliminate these boundary generated gravity inertia waves.

The strategy can be summarized as follows: treat the boundary in a crude manner, but make sure this crudeness does not compromise forecast accuracy in the region of interest; let us call it ‘over-specify and relax’.

2 Boundary strategy for small area models (non-hydrostatic)

What should be the boundary strategy for an operational non-hydrostatic model? The above implementation of the ‘over-specify and relax’ philosophy becomes difficult to justify for non-hydrostatic models. Firstly, the limitations of computer power mean that it is not possible to move the boundaries far enough away to prevent advective errors from contaminating the forecast when rapidly moving depressions are crossing the boundary early in the forecast. Secondly, by definition, gravity waves must now be important. (If not, why go nonhydrostatic?) Therefore, having a boundary damping zone becomes problematic. (How does one distinguish boundary generated ‘noise’ from meteorologically significant gravity wave activity?). Thus, both the ‘slow’ waves and the ‘fast’ waves can now contaminate the forecast.

What should be done? At this time there is no option but to continue with the ‘over-specify and relax’ strategy.

3 Improvements which can be implemented in the immediate future

Within the context of the ‘over-specify and relax’ philosophy there are a number of improvements which can be attempted. (1) A significant improvement is the following. The frequency of the boundary update is an obvious flaw, particularly in the case of nested systems with small innermost integration areas. For maximum accuracy the boundary should be updated every time step of the host model. In a nested environment this is perfectly feasible in principle: run the host model for one time step $\Delta t_h$, at the end of which dump a boundary file for the guest model; stop the host model and run the guest model for $n$ time steps, where $n\Delta t_g = \Delta t_h$, using the boundary files dumped by the host.
model; continue, overwriting the boundary files. The expectation is that the advective waves would now be treated accurately enough to alleviate the need to keep the boundary in the far distance. Additional improvements can be made. (2) Use the host model orography in the relaxation zone. (3) Use weak boundary coupling in the lowest model layers. (4) Tune the relaxation coefficients for fine grids and frequent boundary updates; we now need to damp ‘acoustic’ as well as ‘gravity’ waves. (5) When interpolating the boundary fields to the guest model use a method which produces well-balanced fields. Of course, none of these improvements solves the problem of distinguishing physical gravity waves from those generated by the over-specification of the boundary fields.

4 Improvements which can be implemented in the more distant future

Is there a superior boundary strategy which addresses this issue? In principle, yes, there is a well defined method of treating the boundary accurately based on the ideas of Engquist and Majda, which McDonald, Termonia, and Voitus have been examining. This has been shown to work very well for linear systems for grid point models, and for one dimensional spectral models. A plan for continuing this work for grid point models is given in section 6, and for spectral models in section 7.

5 Essential tool for testing boundary strategies

In HIRLAM we have no rigorous means of testing the impact of changes to the boundary coupling within the HIRLAM framework. Such a procedure does exist. It was described by Baumhefner and Perkey (1982). Let us call it the ‘perfect forecast’ method. The simplest way to set it up is as follows. First, run a (say 48h) forecast over the host area using a fine mesh. This furnishes the ‘perfect forecast’. Second, repeat this forecast over the host area using a coarse mesh. This furnishes the ‘coarse host forecast’. Store all of the prognostic fields at every time step for both these runs. We can now nest a guest area within the host area and test different boundary coupling strategies with rigor, since we have a perfect forecast with which to compare the guest area forecasts. Incidentally, running the guest fine mesh forecast with a boundary update every time step provides us with a definitive test of the coding of the boundary coupling: if the forecast is not identical to the perfect forecast the code is erroneous. See McDonald (1997) for further discussion of what tests should be performed. Probably the best way to set this up is to run the host and guest forecasts simultaneously, as was described in section 2.

6 Strategy for continued research in transparent boundary conditions: grid point models

(1) A first step toward transparent boundary conditions for non-hydrostatic models is to examine the linearized equations of motion in two dimensions (x–z) with a view to deriving boundary conditions which allowed both the gravity waves and the acoustic waves to exit without reflection. (2) So far, transparent boundary conditions have only been derived for linear systems with no vertical shear. This means they cannot be correctly implemented in the full primitive equations, where the advecting velocity varies significantly in the vertical. What is required to overcome this flaw is to do a linear analysis of a system of equations whose advective velocities vary in the vertical, while remaining constant in the horizontal.

This would complete the research into linear systems, prior to a full non-linear implementation.
7 Strategy for continued research in transparent boundary conditions: spectral models

7.1 Strategy
The aim is to investigate the possibilities of developing well-posed LBC’s in a spectral model, specifically for the ALADIN dynamics.

Formulating well-posed LBC’s is a tedious problem, since we essentially need mathematically rigorous formulations as well as numerical stability. This makes the problem much more difficult than physics parameterization and dynamics. Indeed, in the latter the equations are essentially provided by nature while the remaining task is to control the unstable modes. Any well-posed LBC formulation is artificial by its very nature and there is no guideline from nature as to how to treat them. This leaves a wide range of formulations with no guarantee that one of them will work sufficiently well.

The problem is even more tedious for spectral models since in spectral space it is practically impossible to adapt fields locally, in particular at the lateral boundaries.

Research in LBC’s is thus filled with pitfalls. Many of them have already been identified by work carried out in the HIRLAM community by McDonald (2000, 2002, 2003, 2005 and 2006). Therefore his work serves as the benchmark for potential future ideas on spectral well-posed LBC’s.
Two general approaches have now been proposed for solving the ‘spectral problem’: the so-called extrinsic LBC’s (Termonia and Voitus, manuscript) and iterative LBC’s (Voitus, Termonia and Bénard, in preparation).

The ‘Perfectly Matched Layer’ (PML) approach has been demonstrated to be reliable in the sub-gravity case (i.e. when the gravity wave speed is greater than the wind speed velocity). However, in most geophysical applications the supergravity waves provide most of the contribution of the internal gravity waves. This makes PML presently a less attractive candidate for immediate investigation. Nevertheless, we should stay open-minded about it, i.e. monitor the literature, but not immediately focus HIRLAM/ALADIN research on it.
It also becomes even more necessary to implement the approach of Boyd (2005) in ALADIN. We believe that this has to move to highest priority in ALADIN. It will allow all future research on LBC’s to take this as the established solution. This will eliminate any concerns about suspicious accuracy related to the existing spline interpolation, allowing us to focus on the more relevant open issues described below.

7.2 Known pitfalls
A brief list of the pitfalls identified by McDonald are

• The problem that we call “drift” in higher-order LBC conditions. We expect this to show up, independently of the dimensions of the system and of the type equations we are dealing with.

• The “corner problem”: imposing LBC in the corners of a 2D horizontal domain causes problems. The confirms the need for 1D tests to separate this issue from the others. This corner problem will also be present in the spectral model.

• The problem of the trajectory truncation in semi-Lagrangian schemes. The upshot of the past research is that the so-called time interpolation\(^1\) is the best stable candidate for solving this.

\(^1\) in contrast to trajectory truncation and the well-posed buffer zone of McDonald (2000; 2002).
\(^2\) about 20 in 1D spectral-model tests, so too many to make it operationally attractive
However, in its present state in the spectral-model setup, this time interpolation is computed with an iterative procedure which needs many iterations.

- Tests in a realistic setup (with real data) of McDonald (2003) without initialization made the LBC’s inaccurate. This is very worrying, since it suggest that there exists a critical level of spurious gravity waves that can not be supported by the LBC formulation.

In the full 3D model it is difficult to properly diagnose these issues separately. Therefore the proper approach to make progress in this domain is to progressively proceed from simple models, via more sophisticated ones to the full model.

Some issues specific to the realistic equations are:

- The hydrostatic primitive equations
  - The hydrostatic equations are not mathematically well-posed. There is an incompatibility between integrating the hydrostatic equation vertically and handling the LBC’s properly.
  - Problem of the “anomalous” mode associated with the Lorenz grid. Is there a more elegant solution than ‘tweaked Lorenz grid’ of McDonald (2006)?

- Euler NH
  - Presently it is not yet clear how to formulate the LBC’s. Identifying the different waves (acoustic and gravity) leads to expressions in mode space which can not be straightforwardly transformed into a spatial constraint formulated in terms of differential operators, unless one uses approximations. It may not be possible to find approximations valid for both gravity waves and acoustic waves simultaneously.

7.3 Extrinsic LBC’s

The question posed by Termonia and Voitus (2007) is whether the lateral boundary conditions can be generated by means of a numerical scheme that is strictly distinct from the one that is used for the dynamics (SI SL). This is called an extrinsic scheme. We now have evidence in the 1D case that this is true. The 1D tests have even shown that the extrinsic scheme has the additional ‘free benefit’ of solving the Semi-Lagrangian trajectory truncation problem.

The primary aim is to carry this through to more realistic equations and higher dimension. If the validity of this concept is confirmed this may turn out to be useful for various reasons. The most conspicuous one is that one could use an Eulerian scheme at the boundaries, which would eliminate the trajectories altogether.

The disadvantage is that we are on scientifically unexplored territory. Additionally this approach touches the geometrical structure of the model. So one will be forced to revisit the model data flow at some stage.

A possibility that should be explored is to write an extrinsic scheme in the basis of the characteristic values, (Shoucri 2004) which has some potential of providing enhanced stability.

7.4 Iterative spectral LBC’s

Here one builds on existing knowledge to get better stability by iterating the dynamics with corrections at the lateral boundaries. The disadvantage is that LBC’s will be tightly intertwined in the dynamics, so future developments in dynamics will influence the treatment of the LBC’s and viceversa.

Currently, the Achilles heel of this approach is in the convergence of the iterations. Four iterations
were necessary in the 1D tests to get acceptable results, which is discouraging. However, there are still un-investigated choices that might accelerate convergence. This will be the primary subject of study.

7.5 Model progression

Obviously, in full the 3D model, there are too many things that may go wrong at the same time. The only way is to solve the problems by progressively going from simple to more sophisticated models. We propose the following model progression:

- 1D SW with Coriolis.
- 2D SW $x - y$
- 2 layer model
- 2D HPE $x - \sigma$
- 3D

The 1D and 2D linear models have analytical solutions. However, in each case a non-linear version of these laboratory models will be investigated. Clearly, for the non-linear tests a ‘perfect forecast’ setup will have to be developed first. It should be noted that a ‘perfect forecast’ setup exists for the full 3D ALADIN model (work of Masek and Vána).

The search for a more elegant solution than the "tweaked Lorenz" of McDonald (2006b) should be done in the 2D $x - \sigma$ model.

Tests in the shallow-water models should start with zero-th order transparent LBC’s and once these work they should be replaced by first-order ones to test the problem of the drift.

In this model progression it will be checked whether the proposed solutions are consistent with the existing 3D ALADIN dynamics.

For the extrinsic LBC approach this leads to the following road map:

1. Tests in the 1D linear shallow-water model with different extrinsic schemes; tests with bell-shaped feature entering the domain and radiation and adjustment test. This has been done (Termonia and Voitus).
2. The problem of the inaccuracy coming from the different nature of the spectral and the grid point space (specifically, the so-called Q operator in Termonia and Voitus) should be studied. For this ALADIN should first, or in parallel, switch to the approach proposed by Boyd (2005).
3. The problem is now that substepping still has some CFL condition. So the next step is to test whether this approach of the base characteristic approach (Shoucri, 2004) allows to get a solution which is sufficiently practically unconditionally stable, provided one makes a small number of substeps. The same tests in the linear 1D shallow-water model as in point 1 will be used as benchmark.
4. Instead of first testing the non-linear 1D shallow-water model, the next step is to go to the 2D linearized shallow water to see whether (i) extra complications will arise due to the organization of the extrinsic buffer near the boundary, (ii) to check the solution of Shoucri of time splitting to handle the characteristics in 2D, and (iii) to check the corner problem.
The precise sequence of the subsequent research will then depend on the success/failure to solve the above issues.

For the iterative approach,
- the first tests are carried out in the 1D shallow-water model. The main goal is to study alternatives to accelerate the convergence of (i) the iteration in the 1D linear shallow-water model and (ii) the convergence of the solver of the trajectory time interpolation.
- Next one will first go to the non-linear version of the 1D shallow-water model in order to study the inclusion of the non-linear terms in the modified equation within the iterations.

Since convergence is the main issue here, it will then be decided from the outcome of these tests how to proceed to the more sophisticated models.

8 Diagnostics

Currently, the following diagnostics have been used: RMS errors of the forecast fields, mean average divergence and mass budgets. For the non-linear model the only option, to our knowledge, is to carry out ‘perfect forecast’ tests. All aspects of LBC’s should preferably be tested distinctly. For instance, outgoing modes should be tested in radiation experiments to check transparency. Subsequently, one should inject incoming modes to tests whether the meteorological features enter the domain correctly. Both aspects should systematically be tested, preferably separately.

9 Links with research on DFI and MCUF

Recent tests of Termonia and Deckmyn have shown that the Lothar storm has been propagating in the same part of the spectrum as the gravity waves, which are usually filtered by DFI. Also, some study in the context of the monitoring of the coupling-update frequency (MCUF, see Termonia 2003; 2004) in ALADIN has shown that in this case coupling updates with intervals of about 15 to 20 min, i.e. roughly the time step of the coupling model (ARPEGE in this case) is necessary to have a guarantee that the signal is sufficiently well coupled.

10 Discussion

As said, it is mostly in the non-hydrostatic case the well-posed LBC’s could be expected to provide an essential non-hydrostatic contribution by better treating the gravity waves. So probably the most important hurdle to take is to theoretically find a formulation of well-posed LBC’s for the Euler equations, as a sine qua non for all the rest.

In this context, the link with the research on DFI and MCUF should be kept in mind. Also, we believe that the problem of filtering gravity waves (and signal of the most extreme storms) is not quite well understood at present. In the case of the storm, this problem can be dealt with by the MCUF approach. However to couple gravity waves such an approach is not suitable. Stated differently, it may, in the long run turn out scientifically that one will need to use LBC’s updates at the frequency of the coupled host model time step in order to benefit from the non-hydrostatic models.
References


